The Monge-Ampère differential equation and semi-Lagrangian advection schemes

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We discuss a class of semi-Lagrangian advection schemes based on the selection of a contour in a space-time continuum for the integration of a fluid variable PSI. In our numerical examples, the form of the equations resulting from the choice of this specific contour leads to a non-conservative advection technique for PSI. We supplement this technique with a correction based on the satisfaction of the mass continuity equation in which a nonlinear elliptic equation, the Monge-Ampère differential equation, must be solved at each step of the advection process. This approach allows for the selection of larger time steps than those allowed by the Courant-Friedrich-Lewy stability condition (CFL) encountered in Eulerian schemes and provides the overall numerical scheme with an improved set of trajectories satisfying mass continuity equation. A comparative study between Eulerian and semi-Lagrangian methodologies in a variety of flows such as decaying and evolving turbulence as well as a two-dimensional double-layer shear flow will be presented.