Moist processes in the atmosphere:

From simple concepts to sophisticated parameterizations

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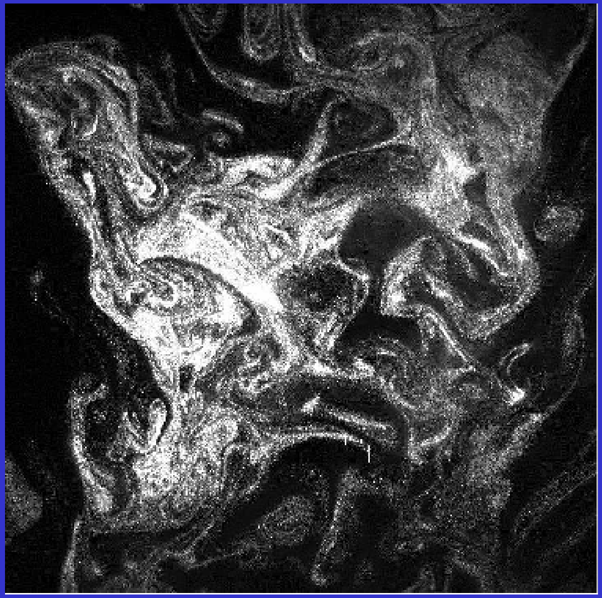
Earth
in visible light

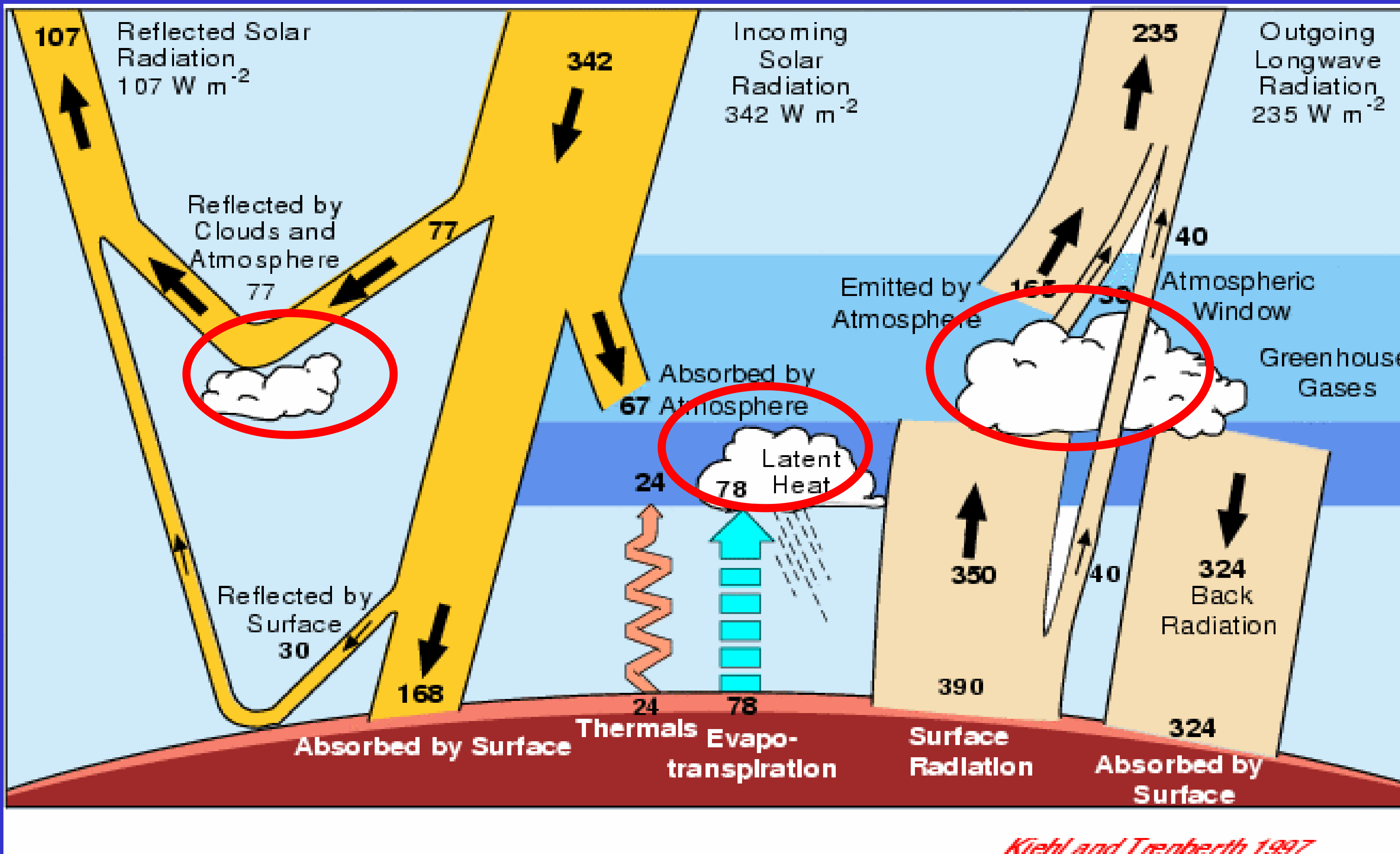
1,000 km

Small cumulus
clouds

Mixing in laboratory
cloud chamber



10 cm



The Earth annual and global mean energy budget

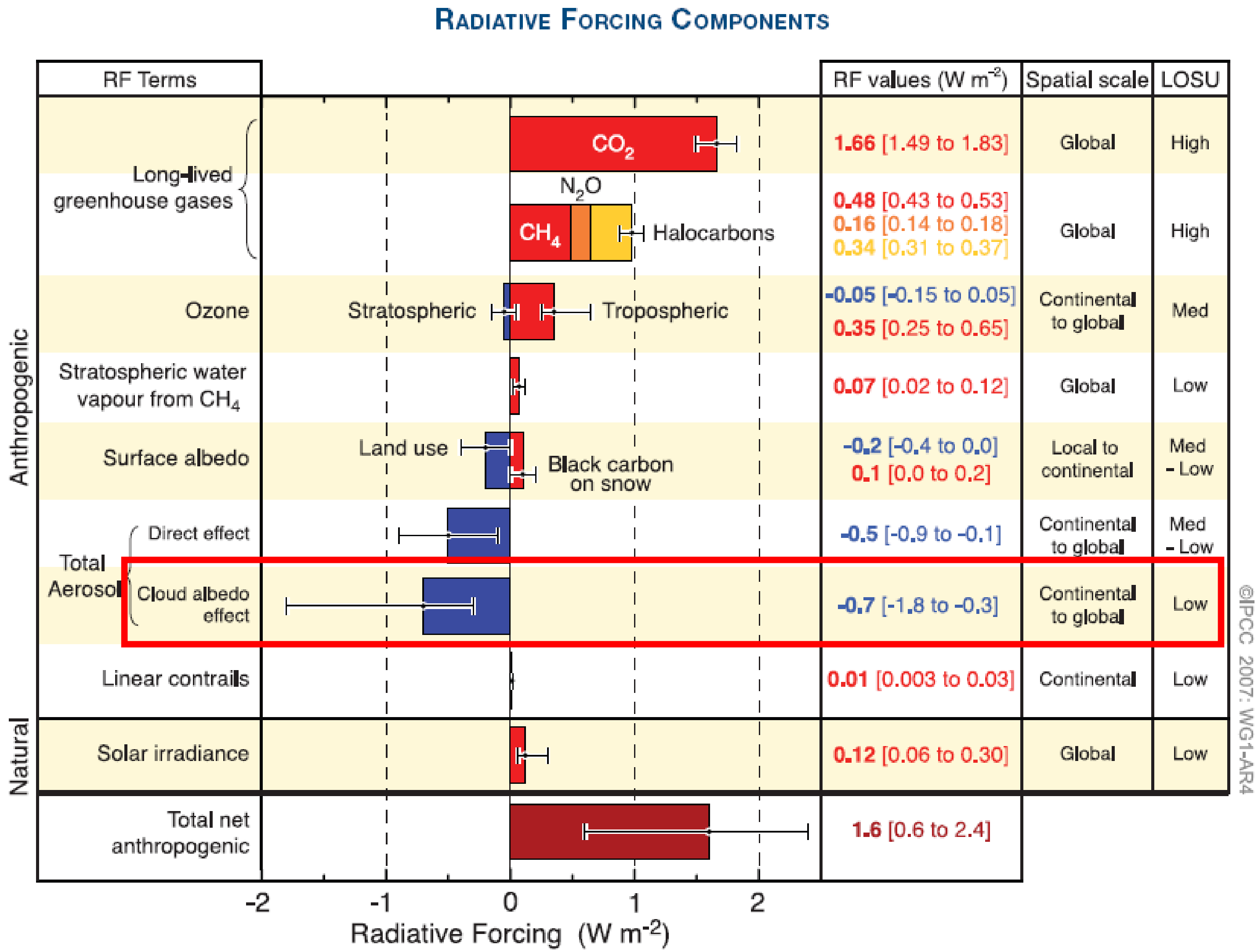


Figure SPM.2. *Global average radiative forcing (RF) estimates and ranges in 2005 for anthropogenic carbon dioxide (CO_2), methane (CH_4), nitrous oxide (N_2O) and other important agents and mechanisms, together with the typical geographical extent (spatial scale) of the forcing and the assessed level of scientific understanding (LOSU). The net anthropogenic radiative forcing and its range are also shown. These require summing asymmetric uncertainty estimates from the component terms, and cannot be obtained by simple addition. Additional forcing factors not included here are considered to have a very low LOSU. Volcanic aerosols contribute an additional natural forcing but are not included in this figure due to their episodic nature. The range for linear contrails does not include other possible effects of aviation on cloudiness. {2.9, Figure 2.20}*

Cloud microphysics 101

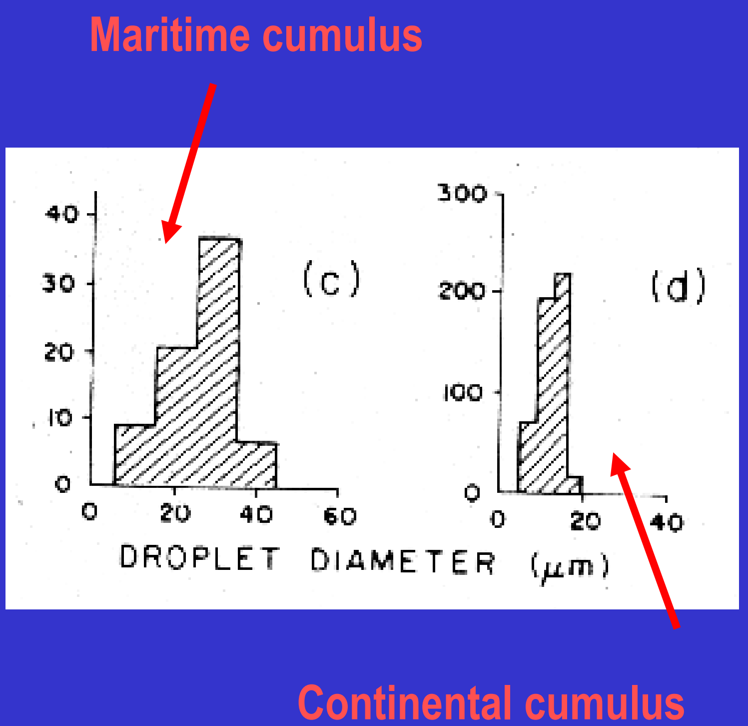
ELEMENTARY CLOUD PHYSICS:

clouds form due to cooling of air (e.g., adiabatic expansion of a parcel of air rising in the atmosphere)

- *condensation:* water vapor ⟶ cloud droplets

heterogeneous nucleation on atmospheric aerosols called Cloud Condensation Nuclei (CCN); typically highly soluble salts (sea salt, sulfates, ammonium salts, nitrates)

only a very small percentage of CCN used by clouds (i.e., water clouds form just above saturation)



ELEMENTARY CLOUD PHYSICS, cont.:

- *formation of ice particles*

heterogeneous nucleation on atmospheric aerosols called Ice-forming Nuclei (IN); dominates for temperatures higher than about -40 deg C (233 K); poorly understood; various modes (contact, deposition, condensation-freezing)

IN are typically silicate particles (clays) or other compounds with crystallographic lattice similar to ice, highly insoluble (contact nucleation) or coated with soluble compound (condensation-freezing)

IN are scarce, their number depends strongly on temperature (typically, 1 per liter at -20 deg C, 10 per liter at -25 deg C).

homogeneous freezing is possible once droplet temperature is smaller than about -40 deg C.

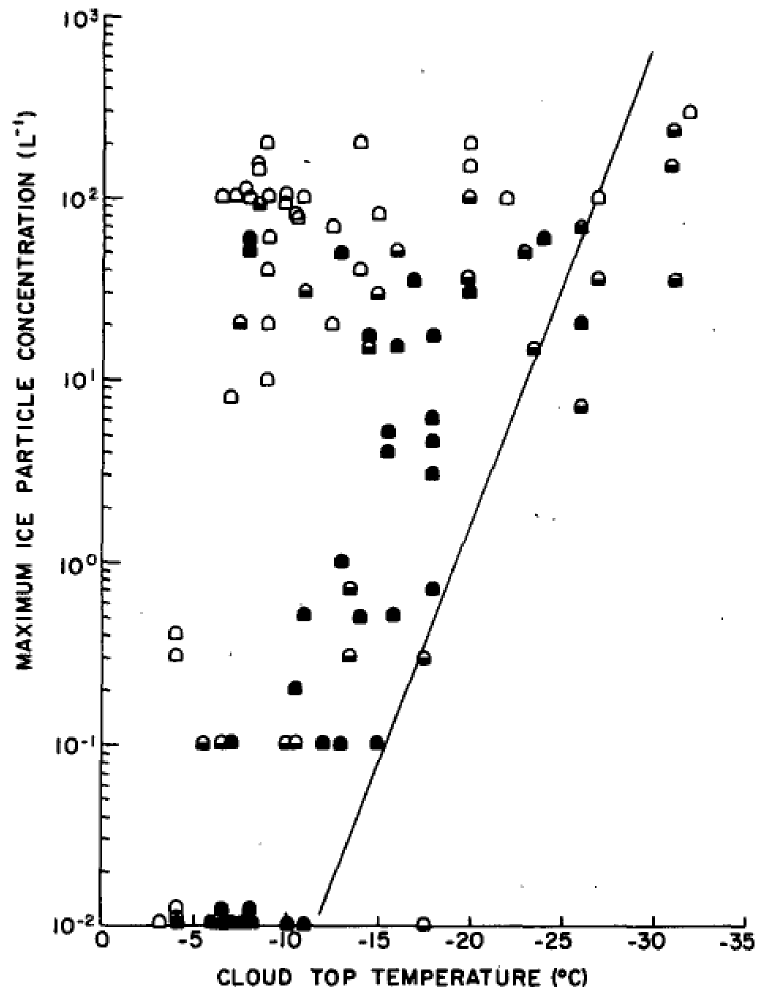


FIG. 2. Measurements of the maximum ice particle concentrations in mature and aging maritime (open humps), continental (closed humps) and transitional (half-open humps) cumuliform clouds. The line represents the concentrations of ice nuclei given by Eq. (1).

From cloud droplets and ice crystals

to precipitation:

WARM RAIN:

⟶ gravitational collision and coalescence between cloud droplets

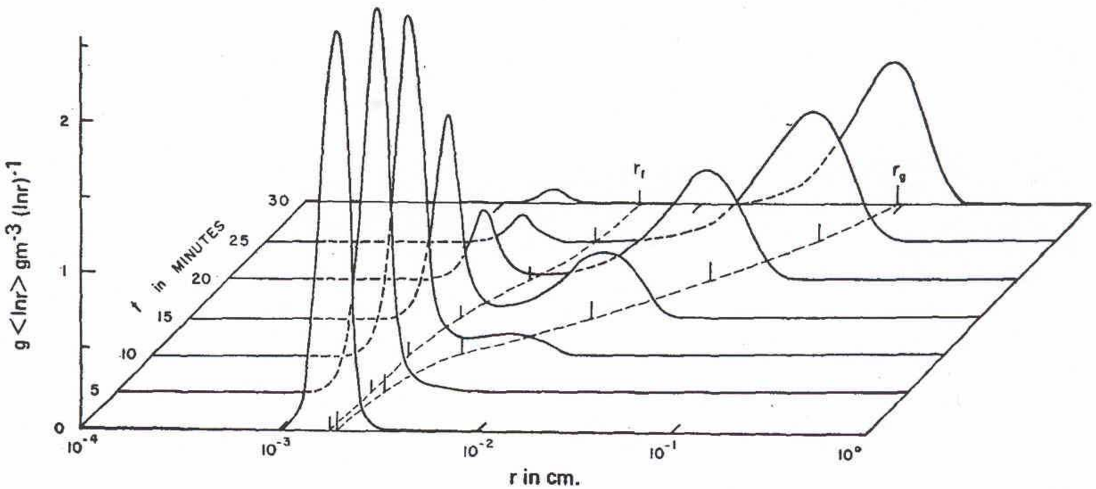


FIG. 5. Time evolution of the initial spectrum for $r_f^0 = 18$ μm, var $x = 0.25$.

Berry and Reinhardt JAS 1974

THE DISTRIBUTION OF RAINDROPS WITH SIZE

By J. S. Marshall and W. McK. Palmer[1]
McGill University, Montreal

(Manuscript received 26 January 1948)

$$N_D = N_0 e^{-\Lambda D}$$

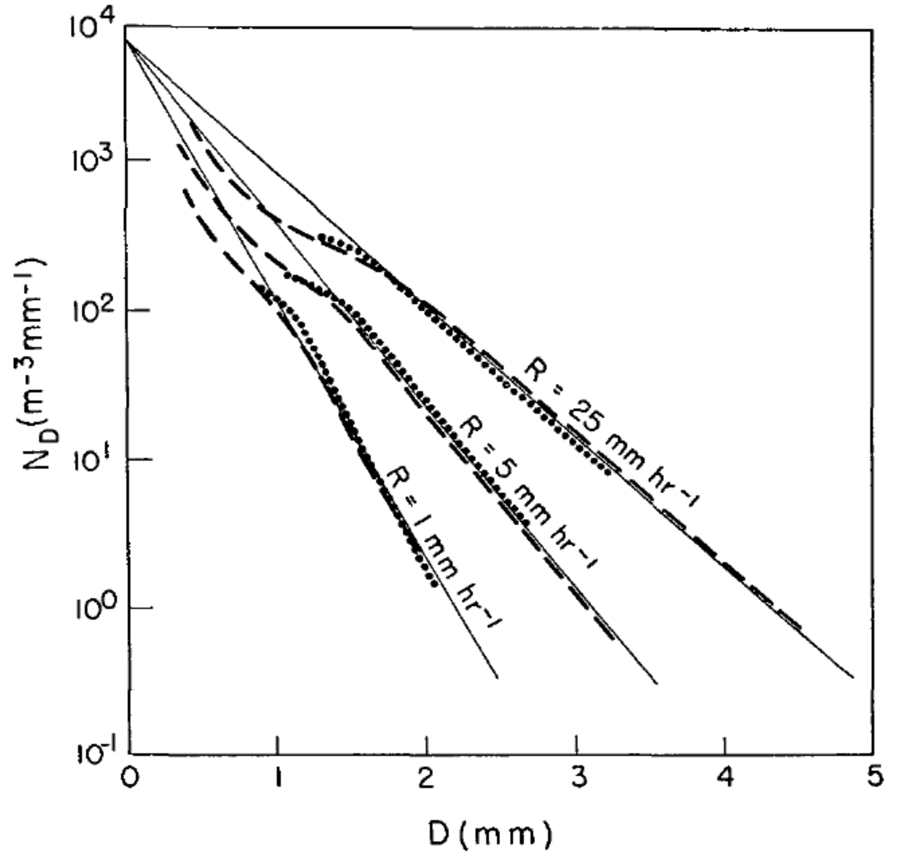


FIG. 2. Distribution function (solid straight lines) compared with results of Laws and Parsons (broken lines) and Ottawa observations (dotted lines).

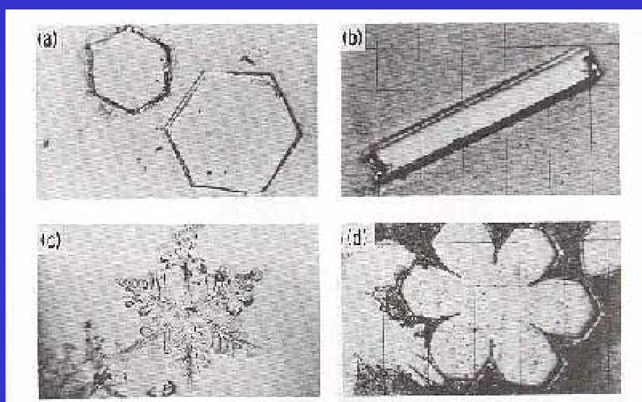
From cloud droplets and ice crystals
to precipitation:

ICE PROCESSES:

⟶ Findeisen-Bergeron process: water vapor pressure at saturation is lower over ice than over water; it follows that once ice crystal is formed from supercooled droplet, it grows rapidly through diffusion of water vapor at the expense of cloud droplets

⟶ riming: falling ice crystal collects supercooled droplets that freeze upon contact (graupel, hail, etc).

Pristine ice crystals, grown by diffusion of water vapor



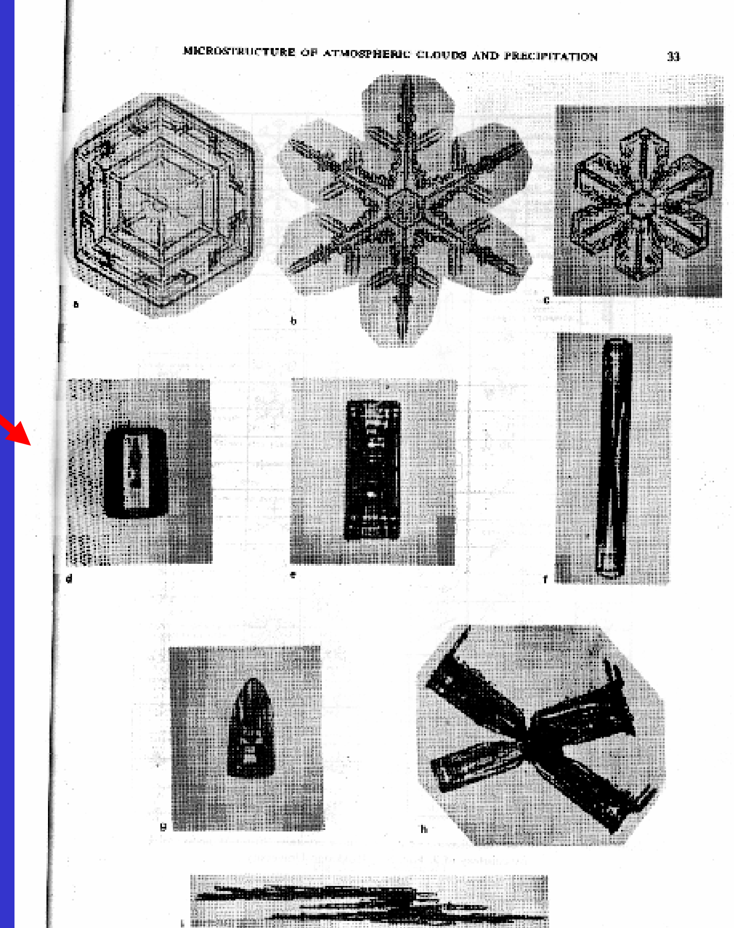
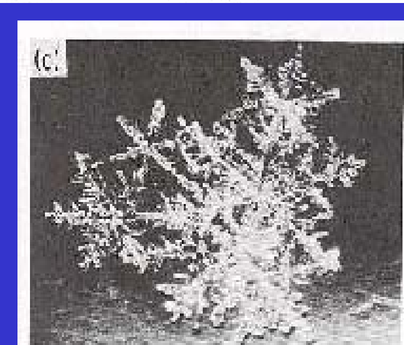
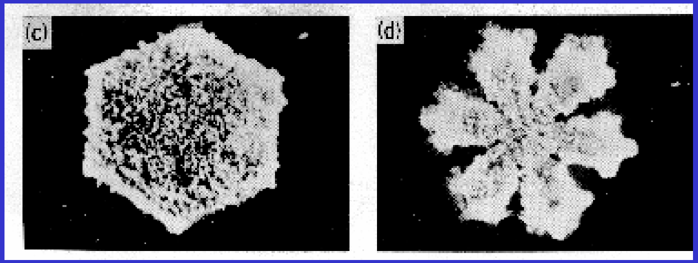


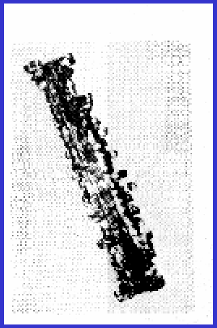
Plate 2. Major shapes of snow crystals: (a) simple plate, (b) dendrite, (c) crystal with broad branches, (d) solid column, (e) hollow column, (f) sheath, (g) bullet, (h) combination of bullets (rosette, Prismenbüschel), (i) combination of needles. (From Nakaya, 1954; by courtesy of Harvard University Press, copyright 1954 by the President and Fellows of Harvard College.)

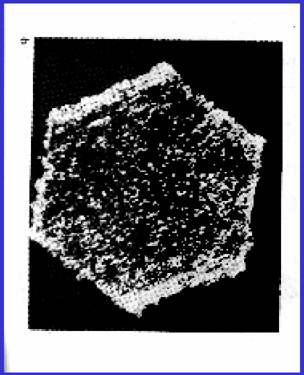
Snowflakes, grown by aggregation

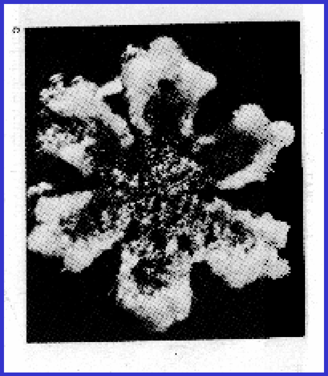


Rimed ice crystals (accretion of supercooled cloud water)

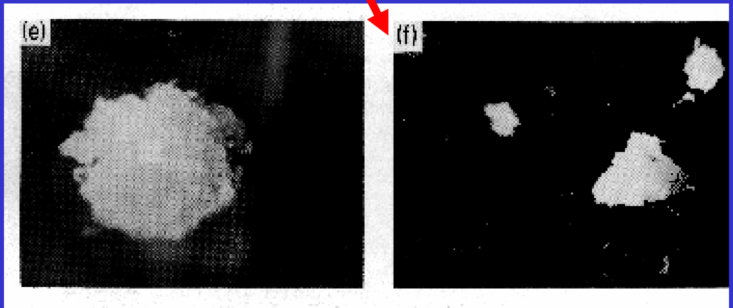






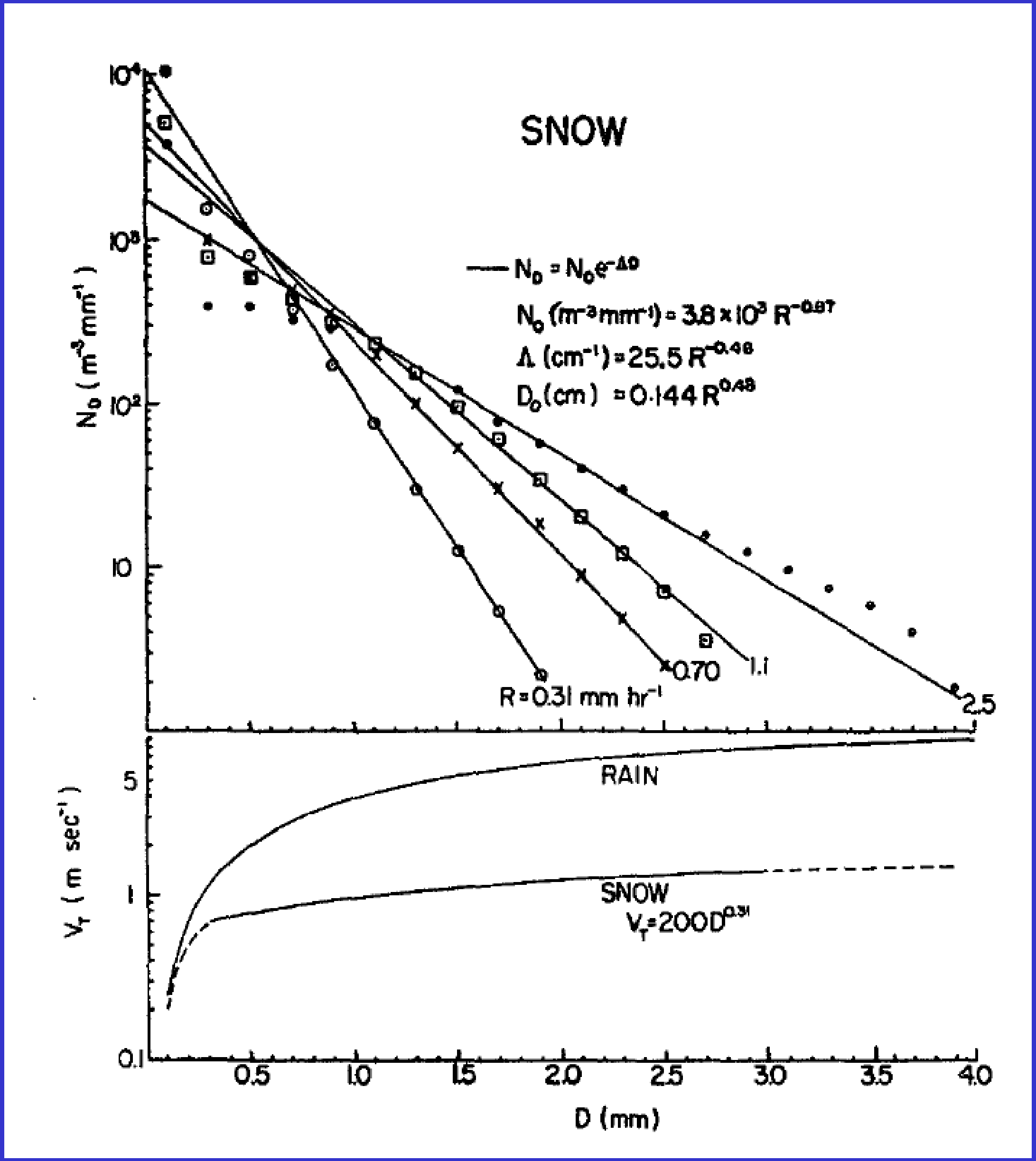


Graupel (heavily rimed ice crystals)



Hail (not to scale)

$$N_D = N_0 e^{-\Lambda D}$$



Gunn and Marshall JAS 1958

Modeling moist processes in the atmosphere:

- Gas dynamics for the air with moisture (i.e., containing water vapor, suspended small cloud particles, falling larger precipitation particles);

- Thermodynamics for the air containing water vapor (i.e., phase changes, latent heating, etc).

Gas dynamics for moist air:

- **Water vapor is a minor constituent:**

  mass loading is typically smaller than 1%; thermodynamic properties (e.g., specific heats etc) only slightly modified;

- **Suspended small particles (cloud droplets, cloud ice):**

  mass loading is typically smaller than a few tenths of 1%, particles are much smaller than the smallest scale of the flow; multiphase approach is not required

- **Precipitation (raindrops, snowflakes, graupel, hail):**

  mass loading can reach a few %, particles are larger than the smallest scale of the flow; multiphase approach needed only for very-small-scale modeling (e.g., DNS).

Thermodynamics:

Moist air is treated as a perfect gas

Phase changes lead to the release of latent heat and formation of condensed (liquid or solid) phase of the water substance (cloud droplets, raindrops, ice crystals, snow, etc)

Condensed phase is treated as continuous medium, i.e., described as density (of cloud droplets, raindrops, etc).

In practice, variables most often used to describe water vapor and condensate are not densities, but mixing ratios, i.e., densities normalized by the air density.

First Law of Thermodynamics:

$$dq = du + p\,dv \tag{1}$$

dq - heat (per unit mass) added to the system
du - increase of internal energy (per unit mass)
$p\,dv$ - work (per unit mass) performed by the system

$$du = c_v\,dT, \quad pv = RT, \quad v = 1/\rho, \quad c_v + R = c_p$$

$$dq = c_p\,dT - \frac{RT}{p}dp \tag{2}$$

Indtroducing *potential temperature* as:

$$\theta = T\left(\frac{p_{oo}}{p}\right)^{R/c_p} \tag{3}$$

where p_{oo}=const (typically 1000 mb), (1) can be written as:

$$d\theta = \frac{\theta}{c_p T}dq \tag{4}$$

$$\frac{d\theta}{dt} = \frac{\theta}{c_p T} S$$

where $S = \frac{dq}{dt}$ in the heat source per unit mass [in J kg^{-1} s^{-1}]

$S = 0$ - adiabatic motions

$S \neq 0$ - motions with diabatic processes (heating due to radiative transfer, phases changes, chemical reactions, etc)

For phase changes of water substance:

$$S = L\frac{dQ}{dt}$$

where L is the latent heat (of condensation, freezing, or sublimation), and $\frac{dQ}{dt}$ is the change of corresponding water mixing ratio

In the spirit of the Boussinesq approximation, moisture and condensate affect gas dynamics equations only though the buoyancy term

$$\frac{d\mathbf{u}}{dt} = -\frac{1}{\rho}\nabla p - g\mathbf{k} + \ldots .(\text{Coriolis, turbulence, etc})$$

$$\rho = \rho_o(z) + \rho'$$

$$p = p_o(z) + p'$$

$$(\rho_o + \rho')\frac{dw}{dt} = -\frac{\partial p_o}{\partial z} - \rho_o g - \frac{\partial p'}{\partial z} - \rho' g + \ldots .$$

$$\frac{d\mathbf{u}}{dt} = -\frac{1}{\rho_o}\nabla p' - g\mathbf{k}\frac{\rho'}{\rho_o} + \ldots .$$

For small-Mach number flows ($|\mathbf{u}| \ll c_s$; c_s - speed of sound):

$$\frac{\rho'}{\rho_o} \approx -\frac{T'}{T_o}$$

$$\frac{d\mathbf{u}}{dt} = -\frac{1}{\rho_o}\nabla p' + g\mathbf{k}\frac{T'}{T_o} + \ldots .$$

Density temperature T_d: the temperature dry air has to have to yield the same density as moist cloudy air

$$T_d = T\frac{1 + q/\epsilon}{1 + q + Q}$$

T - air temperature

q - water vapor mixing ratio ($\sim 10^{-3}$)

Q - condensate mixing ratio (cloud water, rain, ice, snow, etc.; $\sim 10^{-3}$)

$$\epsilon = \frac{R_d}{R_v} \approx 0.622$$

$$T_d \approx T\left[1 + \left(\frac{1}{\epsilon} - 1\right)q - Q\right]$$

$$T_d \approx T\,(1 + 0.61q - Q)$$

T, q and Q – thermodynamics (and much more!)

Modeling of cloud microphysics

Lagrangian versus Eulerian formulation

Lagrangian:

$$\frac{d\Psi}{dt} = S \qquad (1)$$

Eulerian:

$$\frac{\partial \Psi}{\partial t} + \mathbf{u} \cdot \nabla \Psi = S$$

$$\frac{\partial \rho \Psi}{\partial t} + \nabla(\rho \mathbf{u} \Psi) = \rho S \qquad (2)$$

NB: I will often write (1), but in practice always mean (2) to ensure conservation during advective transport…

Continuous medium approach: apply density as the main field variable (density of water vapor, density of cloud water, density of rainwater, etc…)

$$\frac{\partial \rho_v}{\partial t} + \nabla(\rho_v \mathbf{u}) = S \quad \text{or} \quad \frac{d\rho_v}{dt} + \rho_v \nabla \mathbf{u} = S$$

In practice, mixing ratios are typically used. Mixing ratio is the ratio between the density (of water vapor, cloud water…) and the air density.

$$\frac{\partial \rho_a}{\partial t} + \nabla(\rho_a \mathbf{u}) = 0 \quad \text{or} \quad \frac{d\rho_a}{dt} + \rho_a \nabla \mathbf{u} = 0$$

$$\frac{\partial \rho_v}{\partial t} + \nabla(\rho_v \mathbf{u}) = 0 \quad \text{or} \quad \frac{d\rho_v}{dt} + \rho_v \nabla \mathbf{u} = 0$$

$$\text{mixing ratio}: \quad Q = \frac{\rho_v}{\rho_a}$$

$$\frac{dQ}{dt} = \frac{1}{\rho_a}\frac{d\rho_v}{dt} - \frac{\rho_v}{\rho_a^2}\frac{d\rho_a}{dt} \equiv 0$$

Mixing ratios versus specific humidities

$$\frac{\partial \rho_a}{\partial t} + \nabla(\rho_a \mathbf{u}) = 0 \quad \text{or} \quad \frac{d\rho_a}{dt} + \rho_a \nabla \mathbf{u} = 0$$

$$\frac{\partial \rho_v}{\partial t} + \nabla(\rho_v \mathbf{u}) = S \quad \text{or} \quad \frac{d\rho_v}{dt} + \rho_v \nabla \mathbf{u} = S$$

$$\text{mixing ratio :} \quad q = \frac{\rho_v}{\rho_a}$$

$$\frac{dq}{dt} = \frac{S}{\rho_a}$$

$$\text{specific humidity :} \quad Q = \frac{\rho_v}{\rho_v + \rho_a}$$

$$\frac{dQ}{dt} = \frac{\rho_a}{\rho_v + \rho_a} \frac{S}{\rho_v + \rho_a}$$

And we also need equation for the temperature. If only phase changes are included, then potential temperature equation is:

$$\frac{d\theta}{dt} = \frac{L\theta}{c_p T}\frac{dq}{dt}$$

L – latent heat (of condensation, freezing, or sublimation)

$\frac{dq}{dt}$ – change of corresponding condensate mixing ratio

Modeling of cloud microphysics: solving a system of PDEs (advection/diffusion type) coupled through the source terms:

$$\frac{d\theta}{dt} = S_\theta$$

$$\frac{dq_v}{dt} = S_{q_v}$$

for $i = 1, N$:

$$\frac{dq_c^{(i)}}{dt} = S^{(i)}$$

θ - potential temperature
q_v - *water vapor* mixing ratio
$q_c^{(i)}$ - *condensed water* mixing ratios
$S^{(i)}$ - sources/sinks for condensed water (phase changes, transfer from one category to another, sedimentation, etc.)

I. Bulk models

BULK MODEL OF CONDENSATION:

$$\frac{d\theta}{dt} = \frac{L_v \theta}{c_p T} C_d$$

$$\frac{dq_v}{dt} = -C_d$$

$$\frac{dq_c}{dt} = \quad C_d$$

θ - potential temperature
q_v - *water vapor* mixing ratio
q_c - *cloud water* mixing ratio
L_v - latent heat of condensation/evaporation
C_d - condensation rate
Note: θ/T function of pressure only ($\approx \theta_o/T_o$)

C_d is defined such that cloud is always at saturation, which is a very good approximation:

$$q_c = 0 \quad \text{if} \quad q_v < q_{vs}$$

$$q_c > 0 \quad \text{only if} \quad q_v = q_{vs}$$

where $q_{vs}(p, T) \approx 0.622 \frac{e_s(T)}{p}$ is the water vapor mixing ratio at saturation

If $\overline{\theta}/\overline{T} = \text{const}$ (shallow convection approximation)

$$\frac{d\theta}{dt} = \frac{L_v \overline{\theta}}{c_p \overline{T}} C_d$$

$$\frac{dq_v}{dt} = -C_d$$

$$\frac{dq_c}{dt} = C_d$$

→

$$\frac{d\theta_I}{dt} = 0$$

$$\frac{dQ}{dt} = 0$$

θ_I is one of the two:

$\theta_e = \theta + \frac{L_v \overline{\theta}}{c_p \overline{T}} q_v$ - equivalent potential temperature

$\theta_l = \theta - \frac{L_v \overline{\theta}}{c_p \overline{T}} q_c$ - liquid water potential temperature

$Q = q_v + q_c$ - total water mixing ratio

$$\frac{d\theta}{dt} = \frac{L_v\bar{\theta}}{c_p\bar{T}}C_d$$

$$\frac{dq_v}{dt} = -C_d$$

$$\frac{dq_c}{dt} = C_d$$

this really implies this...

$$\frac{\partial \rho_o \theta}{\partial t} + \nabla \cdot (\rho_o \theta \mathbf{u}) = \rho_o \frac{L\bar{\theta}}{c_p\bar{T}} C_d$$

$$\frac{\partial \rho_o q_v}{\partial t} + \nabla \cdot (\rho_o q_v \mathbf{u}) = -\rho_o C_d$$

$$\frac{\partial \rho_o q_c}{\partial t} + \nabla \cdot (\rho_o q_c \mathbf{u}) = \rho_o C_d,$$

Time stepping is centered in time:

$$\frac{D\psi}{Dt} = F_c$$

$$\psi^{n+1} = \left(\psi^n + \frac{1}{2}\Delta t F_c^n\right)_o + \frac{1}{2}\Delta t F_c^{n+1}$$

F_c^{n+1} derived from requirement of exact saturation after adjustment (provided that there is enough cloud water for evaporation)

For studies of boundary-layer clouds, liquid water potential temperature and total water are often used:

$$\theta_l = \theta - \frac{L_v \overline{\theta}}{c_p \overline{T}} q_c \text{ - liquid water potential temperature}$$

$$Q = q_v + q_c \text{ - total water mixing ratio}$$

Need to diagnose Θ, q_v and q_c from Θ_l and Q to define buoyancy…

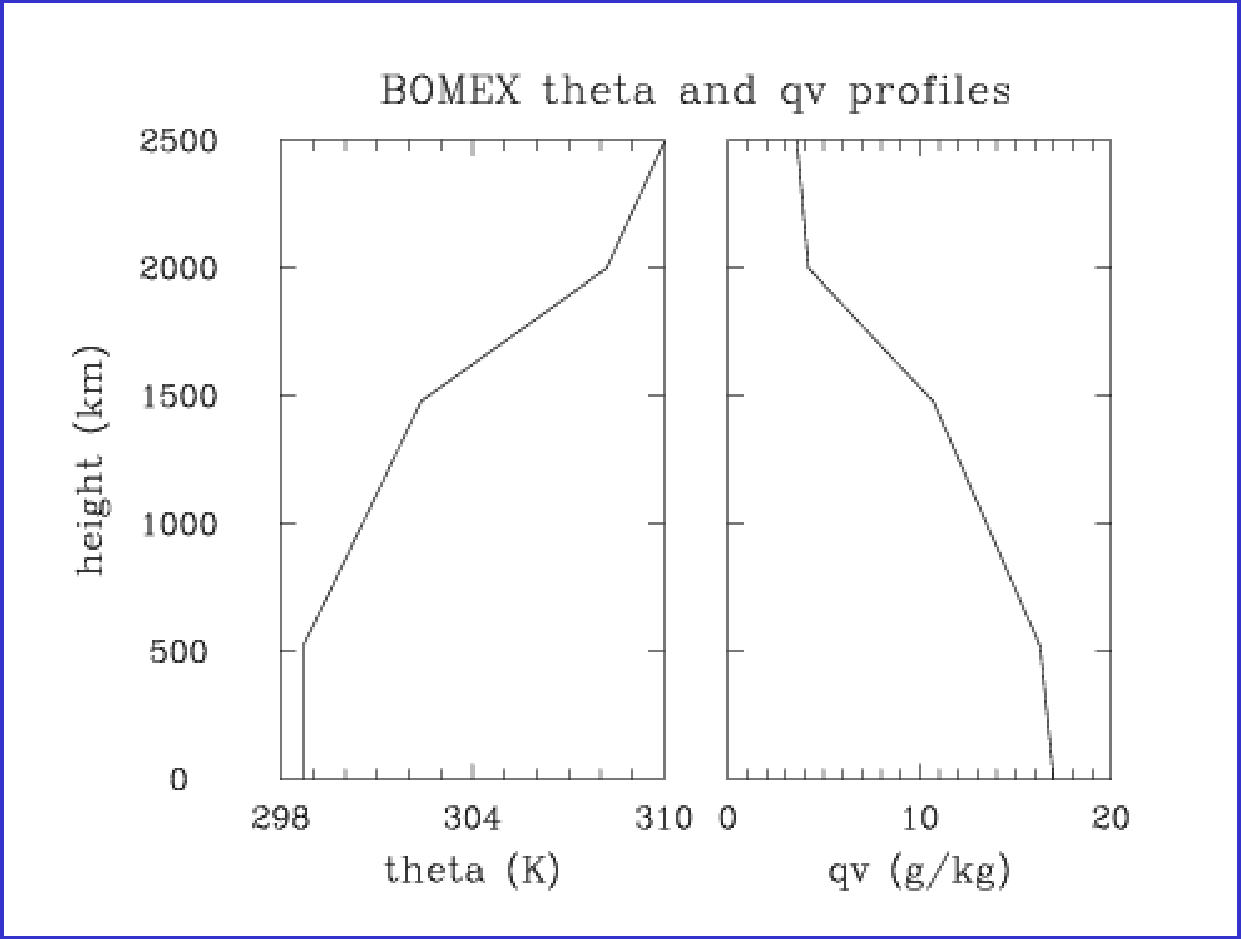
hint: if qc=0: $\Theta_l = \Theta$ and q_v=Q

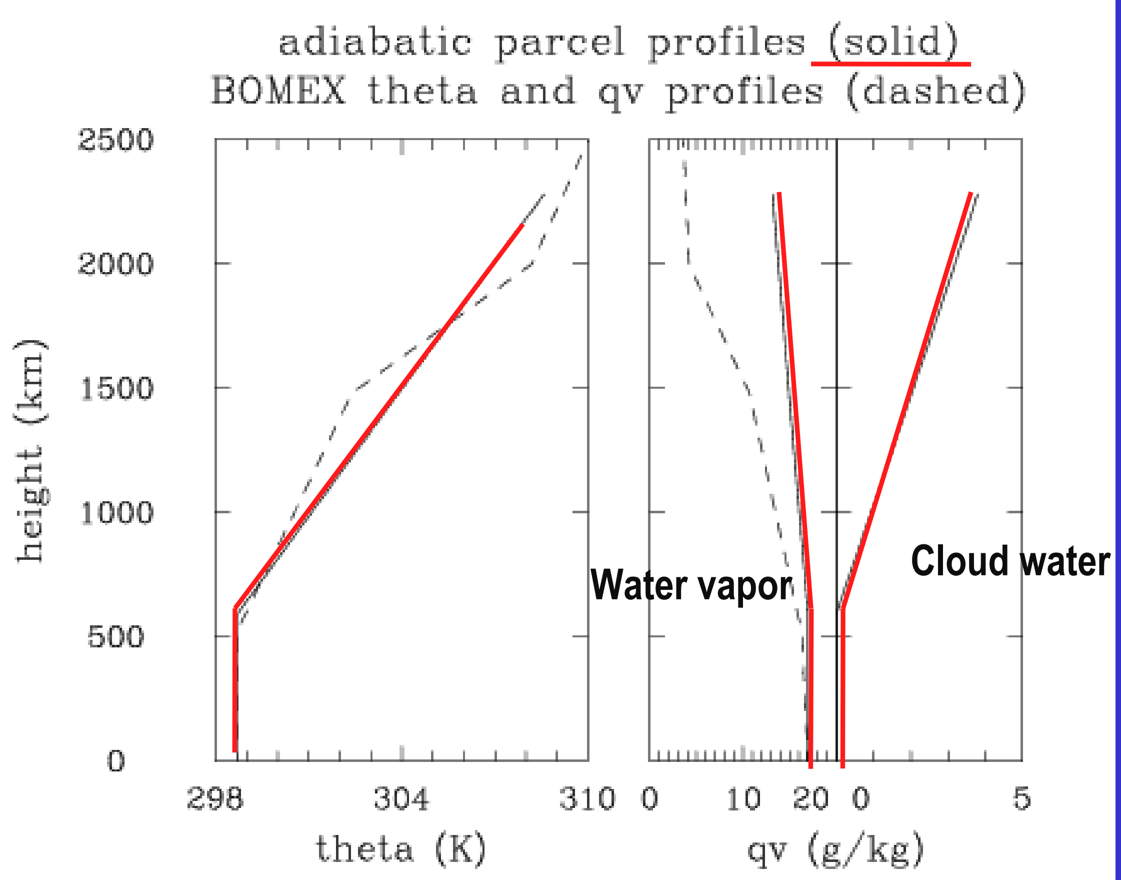
Practical example: rising adiabatic parcel

$$\frac{d\theta}{dt} = \frac{L_v \theta}{c_p T} C_d$$

$$\frac{dq_v}{dt} = -C_d$$

$$\frac{dq_c}{dt} = C_d$$




adiabatic parcel profiles (solid)
BOMEX theta and qv profiles (dashed)
2500
2000
1500
1000
500
0
height (km)
298
304
310
theta (K)
0
10
20
0
5
qv (g/kg)
Water vapor
Cloud water

$$\frac{d\theta}{dt} = \frac{L_v\bar{\theta}}{c_p\bar{T}} C_d$$

$$\frac{dq_v}{dt} = -C_d$$

$$\frac{dq_c}{dt} = C_d$$

this really implies this...

$$\frac{\partial \rho_o \theta}{\partial t} + \nabla \cdot (\rho_o \theta \mathbf{u}) = \rho_o \frac{L\bar{\theta}}{c_p\bar{T}} C_d$$

$$\frac{\partial \rho_o q_v}{\partial t} + \nabla \cdot (\rho_o q_v \mathbf{u}) = -\rho_o C_d$$

$$\frac{\partial \rho_o q_c}{\partial t} + \nabla \cdot (\rho_o q_c \mathbf{u}) = \rho_o C_d,$$

Monotonicity and the MPDATA advection scheme

1st pass – donor cell
2nd pass – antidiffusive flux (applied using donor-cell scheme)

- standard version: sign-preserving
- FCT version: monotonicity enforced (limiters come from CFL criteria)

$$\frac{d\psi}{dt} = 0$$

$$\psi^{n+1} = (\psi^n)_o$$

Monotonicity and the MPDATA advection scheme

1st pass – donor cell
2nd pass – antidiffusive flux (applied using donor-cell scheme)

- standard version: sign-preserving
- FCT version: monotonicity enforced (limiters come from CFL criteria)

$$\frac{d\psi}{dt} = F$$

$$\psi^{n+1} = \left(\psi^n + \frac{1}{2}\,\Delta t\;F^n\right)_o + \frac{1}{2}\,\Delta t\;F^{n+1}$$

Monotonicity and the MPDATA advection scheme for the coupled system:

$$\frac{d\mathbf{\Psi}}{dt} = \boldsymbol{F}(\mathbf{\Psi})$$

$$\mathbf{\Psi}_{\mathrm{i}}^{n+1} = \left(\boldsymbol{\psi}^n + \frac{1}{2}\,\Delta t\,\boldsymbol{F}^n\right)_o + \frac{1}{2}\,\Delta t\,\mathbf{F}^{n+1}$$

$$\frac{\partial \rho_o \theta}{\partial t} + \nabla \cdot (\rho_o \theta \mathbf{u}) = \rho_o \frac{L\bar{\theta}}{c_p \bar{T}} C_d$$

$$\frac{\partial \rho_o q_v}{\partial t} + \nabla \cdot (\rho_o q_v \mathbf{u}) = -\rho_o C_d$$

$$\frac{\partial \rho_o q_c}{\partial t} + \nabla \cdot (\rho_o q_c \mathbf{u}) = \rho_o C_d,$$

analytic solution

$$\frac{\partial \rho_o \theta}{\partial t} + \nabla \cdot (\rho_o \theta \mathbf{u}) = \rho_o \frac{L\bar{\theta}}{c_p \bar{T}} C_d$$

$$\frac{\partial \rho_o q_v}{\partial t} + \nabla \cdot (\rho_o q_v \mathbf{u}) = -\rho_o C_d$$

$$\frac{\partial \rho_o q_c}{\partial t} + \nabla \cdot (\rho_o q_c \mathbf{u}) = \rho_o C_d,$$

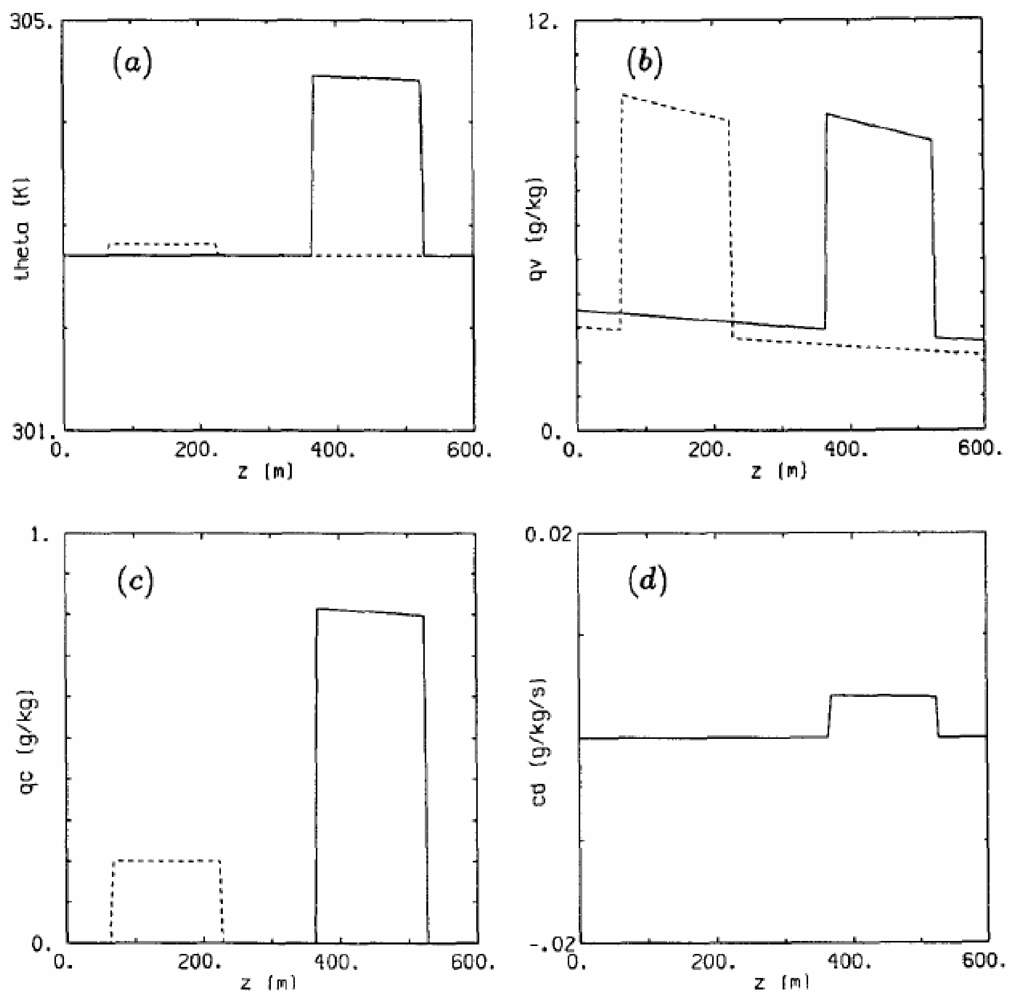


FIG. 1. Analytic solution for the one-dimensional, idealized advection–condensation problem. The dashed and the solid lines represent, respectively, initial conditions and final results for the potential temperature (a), the water vapor mixing ratio (b), the cloud water mixing ratio (c), and the condensation rate (d).

Grabowski and Smolarkiewicz *MWR* 1990

donor-cell solution

$$\frac{\partial \rho_o \theta}{\partial t} + \nabla \cdot (\rho_o \theta \mathbf{u}) = \rho_o \frac{L\bar{\theta}}{c_p \bar{T}} C_d$$

$$\frac{\partial \rho_o q_v}{\partial t} + \nabla \cdot (\rho_o q_v \mathbf{u}) = -\rho_o C_d$$

$$\frac{\partial \rho_o q_c}{\partial t} + \nabla \cdot (\rho_o q_c \mathbf{u}) = \rho_o C_d,$$

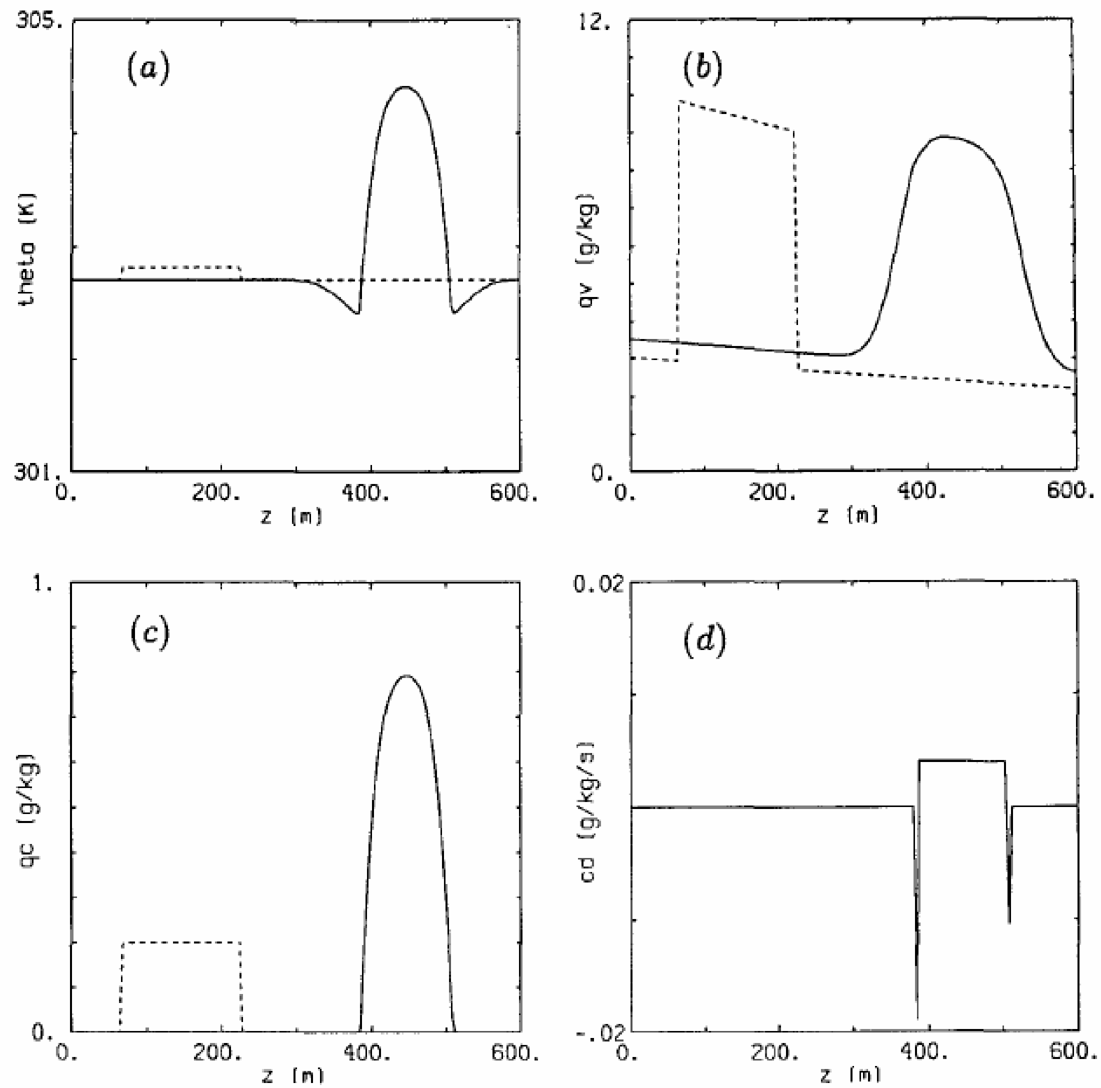


FIG. 2. Numerical solution for the test problem using the fractional-time-steps method with the donor cell advection scheme.

Grabowski and Smolarkiewicz *MWR* 1990

iord=2
monotone
MPDATA

$$\frac{\partial \rho_o \theta}{\partial t} + \nabla \cdot (\rho_o \theta \mathbf{u}) = \rho_o \frac{L\bar{\theta}}{c_p \bar{T}} C_d$$

$$\frac{\partial \rho_o q_v}{\partial t} + \nabla \cdot (\rho_o q_v \mathbf{u}) = -\rho_o C_d$$

$$\frac{\partial \rho_o q_c}{\partial t} + \nabla \cdot (\rho_o q_c \mathbf{u}) = \rho_o C_d,$$

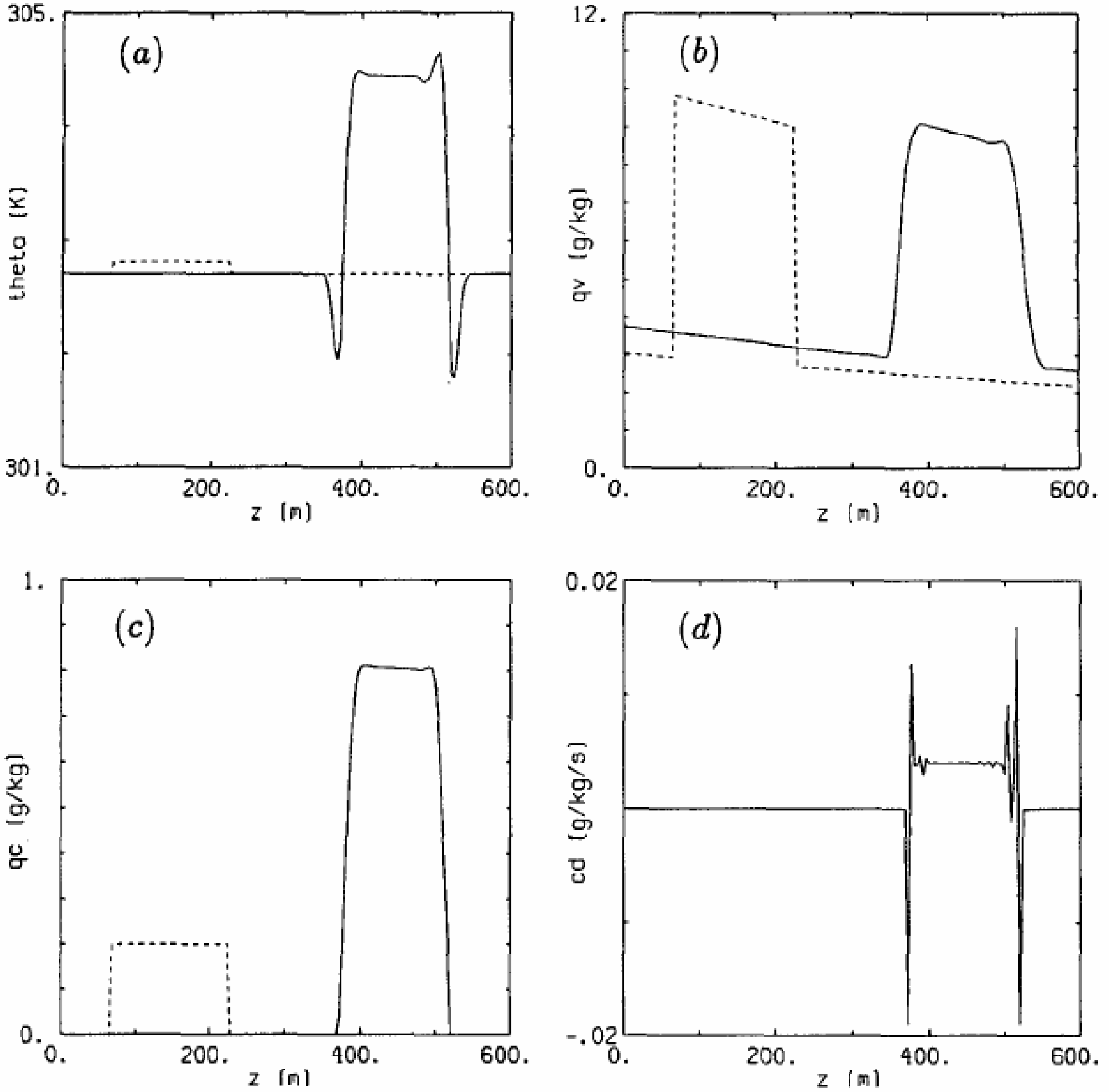


FIG. 4. As in Fig. 3 but with the nonoscillatory MPDATA.

Grabowski and Smolarkiewicz *MWR* 1990

iord=2 custom-designed RH limiter

$$\frac{\partial \rho_o \theta}{\partial t} + \nabla \cdot (\rho_o \theta \mathbf{u}) = \rho_o \frac{L\bar{\theta}}{c_p \bar{T}} C_d$$

$$\frac{\partial \rho_o q_v}{\partial t} + \nabla \cdot (\rho_o q_v \mathbf{u}) = -\rho_o C_d$$

$$\frac{\partial \rho_o q_c}{\partial t} + \nabla \cdot (\rho_o q_c \mathbf{u}) = \rho_o C_d,$$

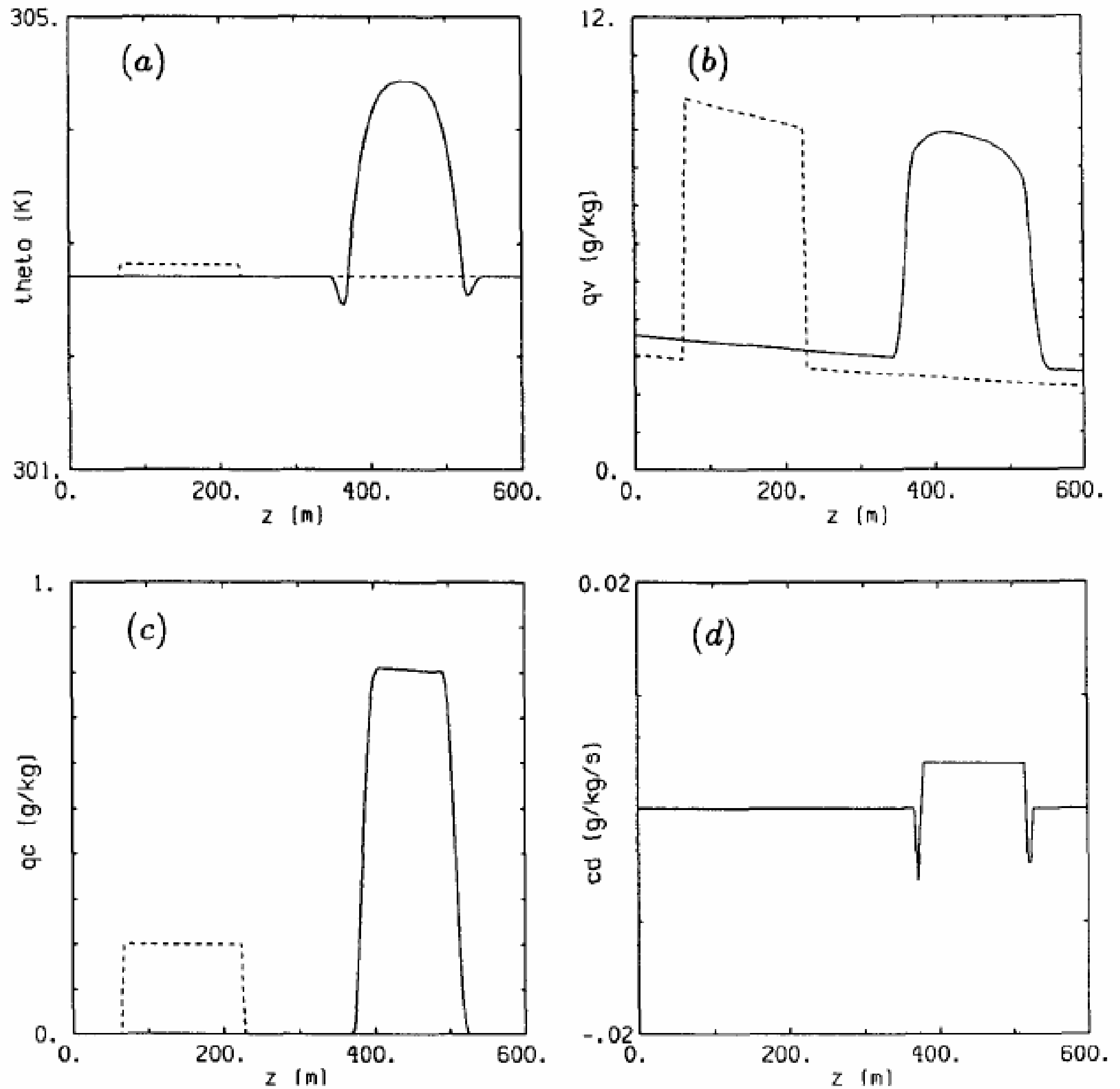


FIG. 5. As in Fig. 3 but with the nonoscillatory advection–condensation scheme [limiting coefficients in the FCT procedure as in (A22)].

Grabowski and Smolarkiewicz *MWR* 1990

iord=2, monotone MPDATA, invariant variables

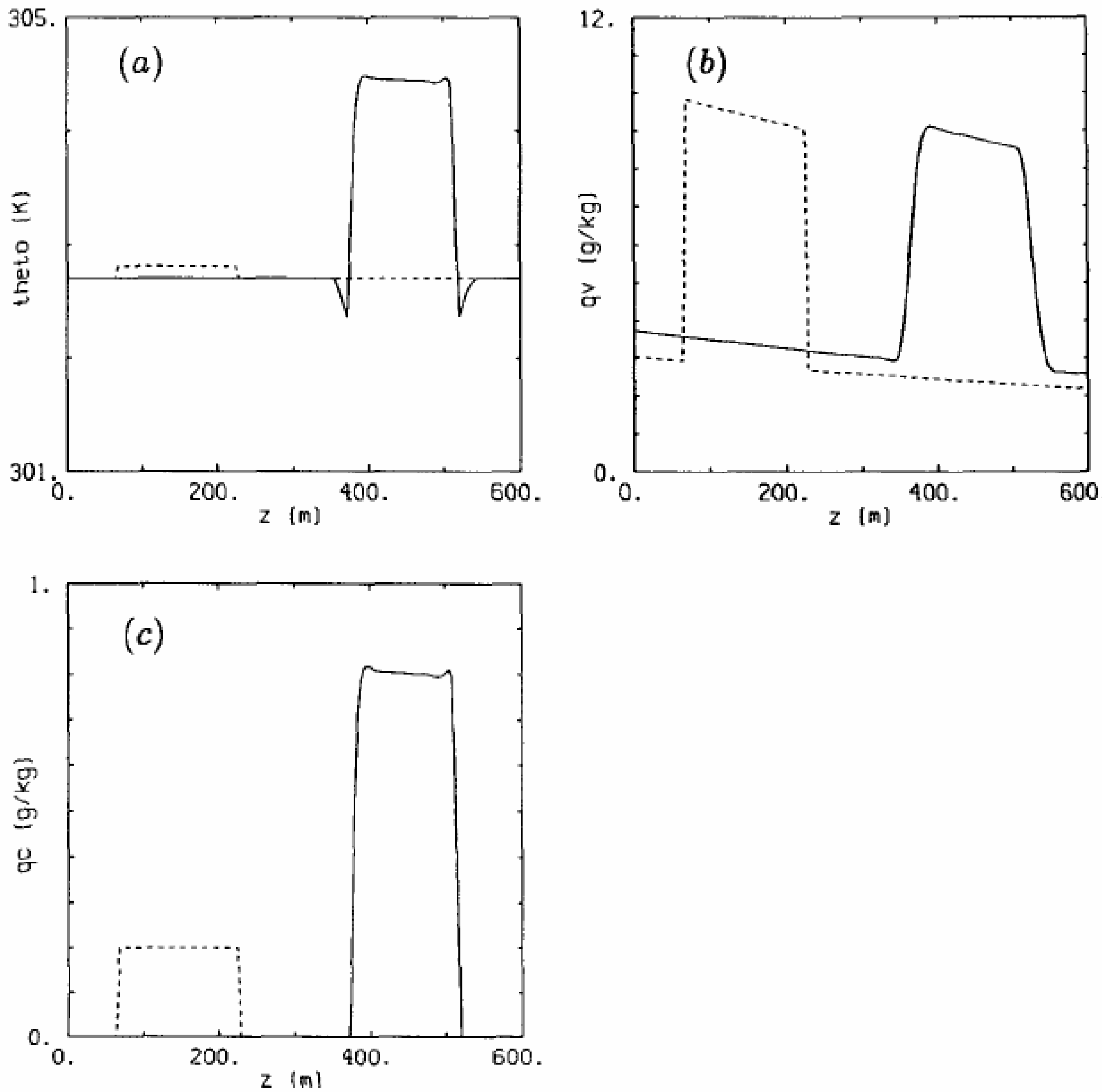


FIG. 7. As in Fig. 6 but with the nonoscillatory MPDATA.

$$\frac{d\theta_I}{dt} = 0$$

$$\frac{dQ}{dt} = 0$$

Grabowski and Smolarkiewicz *MWR* 1990

iord=2, monotone MPDATA, invariant variables with custom-designed limiter for total water

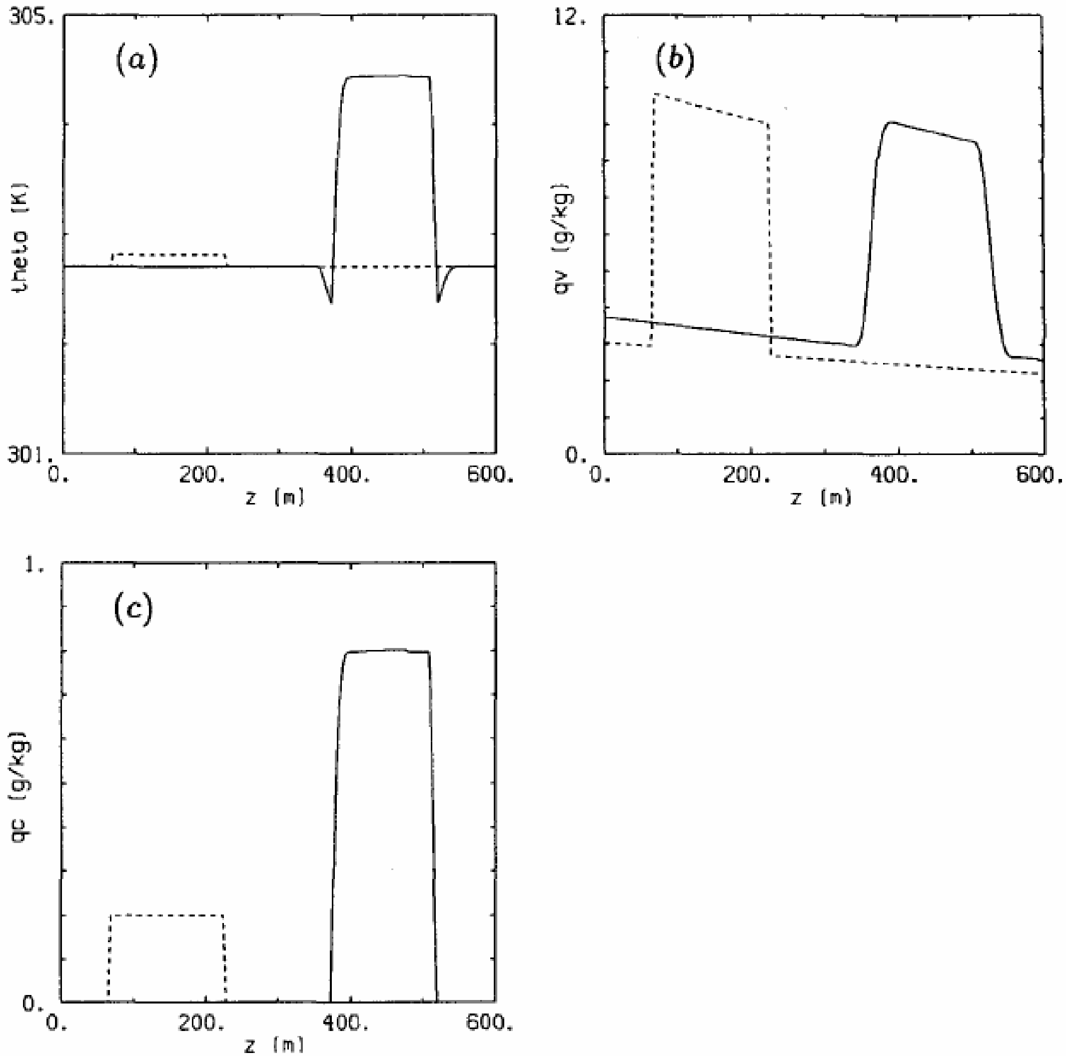


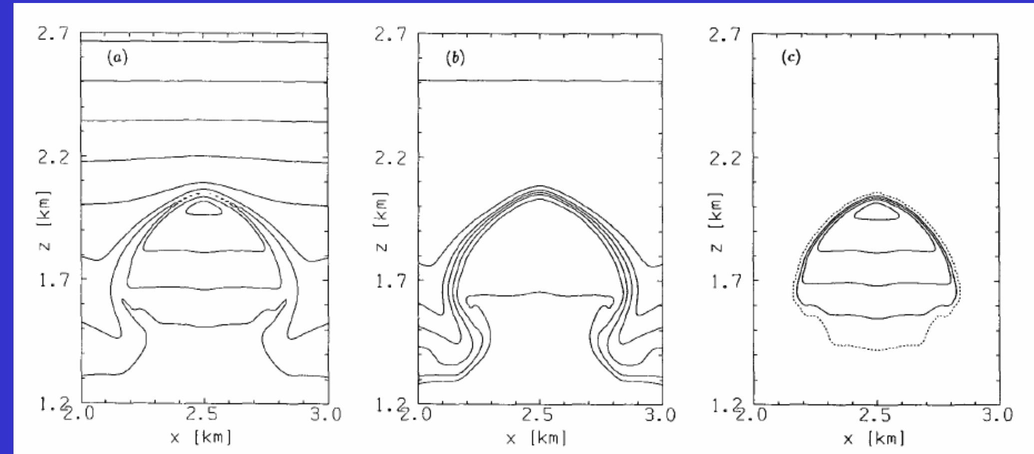
FIG. 8. As in Fig. 6 but with the monotone advection–condensation scheme [nonoscillatory MPDATA with the limiter (B4) for total water substance inside cloudy air].

$$\frac{d\theta_I}{dt} = 0$$

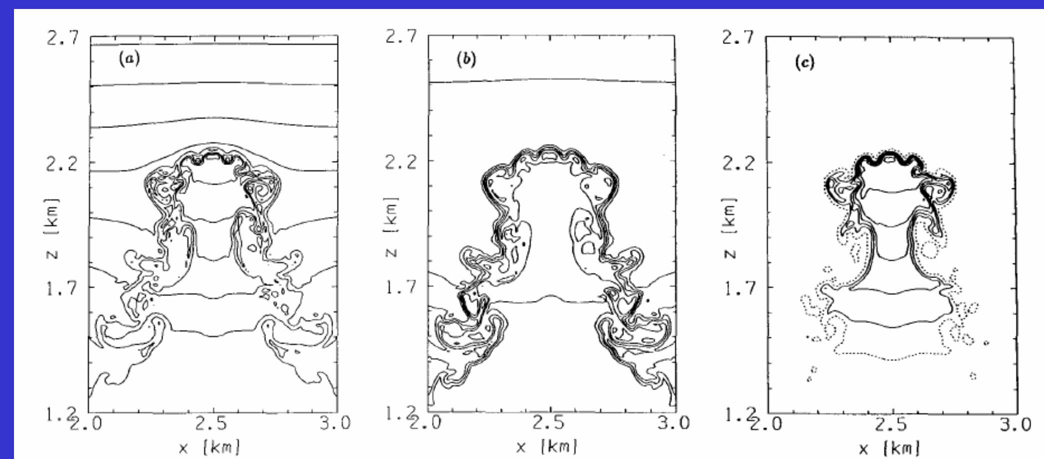
$$\frac{dQ}{dt} = 0$$

Grabowski and Smolarkiewicz *MWR* 1990

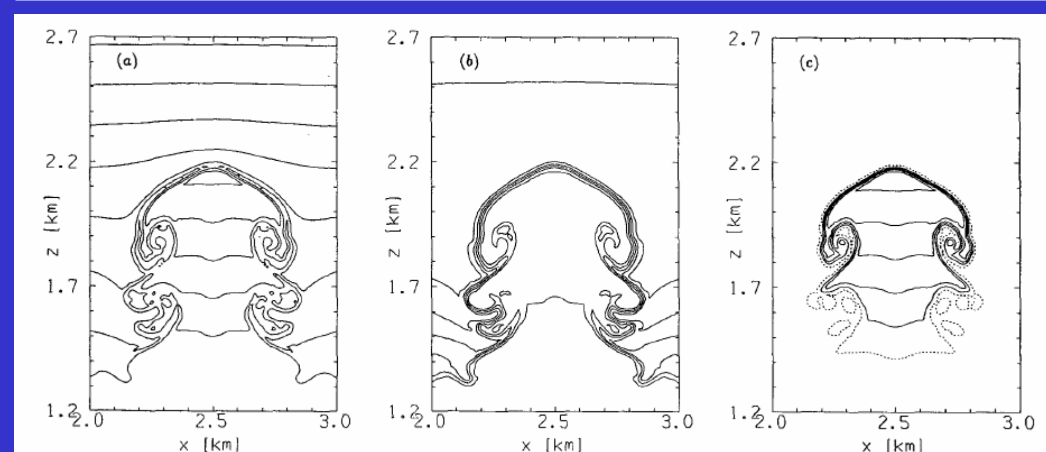
Θ, q_v, q_c; iord=1 (donor-cell)



Θ, q_v, q_c; iord=2; non-FCT



Θ_l, Q; iord=2, custom FCT



Grabowski and Smolarkiewicz *MWR* 1990

Evaporation near cloud edge: see Grabowski and Morrison (*MWR* 2008) for an approach for the bulk double moment scheme…

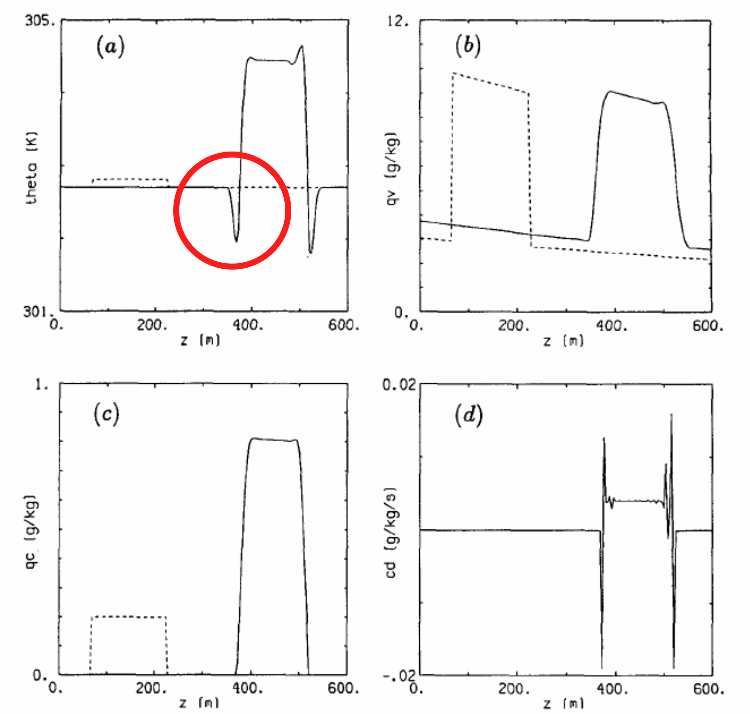


FIG. 4. As in Fig. 3 but with the nonoscillatory MPDATA.

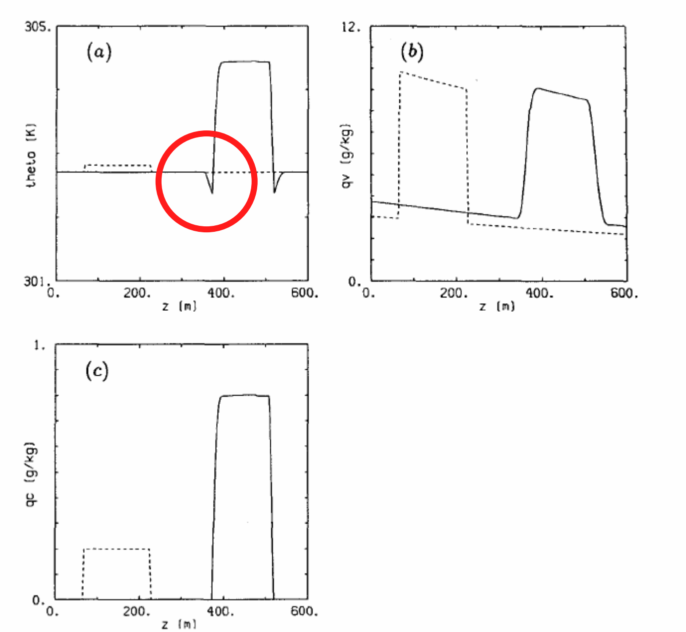


FIG. 8. As in Fig. 6 but with the monotone advection–condensation scheme [nonoscillatory MPDATA with the limiter (B4) for total water substance inside cloudy air].

WARM RAIN BULK MODEL (Kessler 1969):

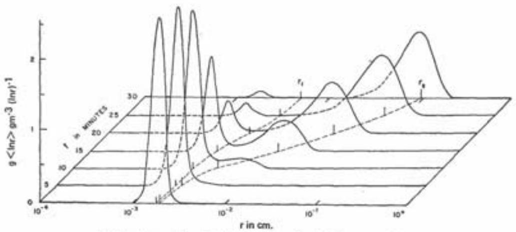


FIG. 5. Time evolution of the initial spectrum for $r_f^0 = 18\ \mu m$, var $x = 0.25$.

$$\frac{d\theta}{dt} = \frac{L_v \theta}{c_p T}(C_d - EVAP)$$

$$\frac{dq_v}{dt} = -C_d + EVAP$$

$$\frac{dq_c}{dt} = C_d - AUT - ACC$$

$$\frac{dq_r}{dt} = \frac{1}{\rho}\frac{\partial}{\partial z}(\rho q_r v_t) + AUT + ACC - EVAP$$

θ - potential temperature
q_v - *water vapor* mixing ratio
q_c - *cloud water* mixing ratio
q_r - *rain water* mixing ratio
C_d - condensation rate
$EVAP$ - rain evaporation rate
AUT - "autoconversion" rate: $q_c \rightarrow q_r$
ACC - accretion rate: $q_c, q_r \rightarrow q_r$

$v_t(q_r)$ - rain terminal velocity (typically derived by assuming a drop size distribution; e.g., the Marshall-Palmer distribution $N(D) = N_o exp(-\Lambda D)$, $N_o = 10^7$ m^{-4}).

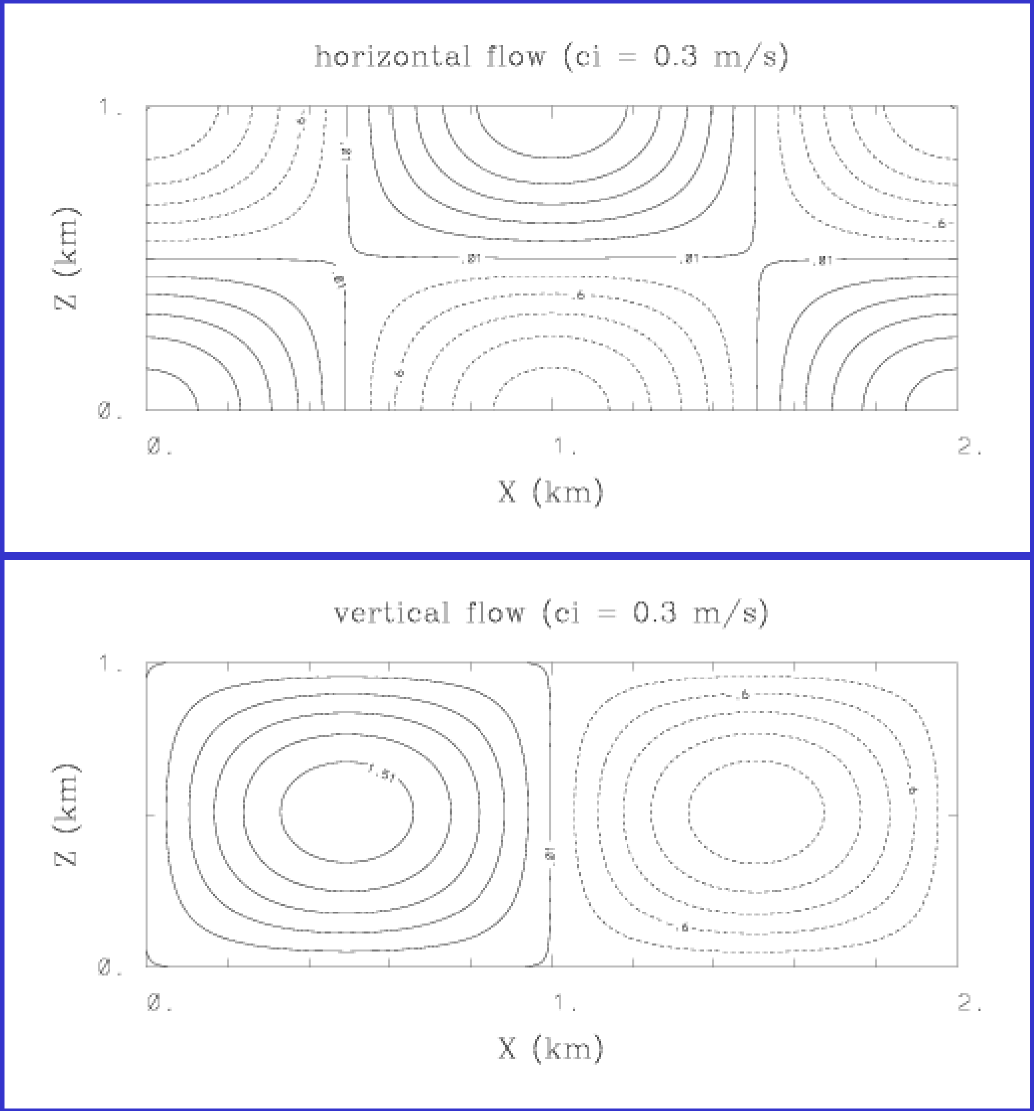
$$\frac{\partial \rho_o \theta}{\partial t} + \nabla \cdot (\rho_o \mathbf{u} \theta) = \frac{L_v \theta_e}{c_p T_e}(C_d - EVAP)$$

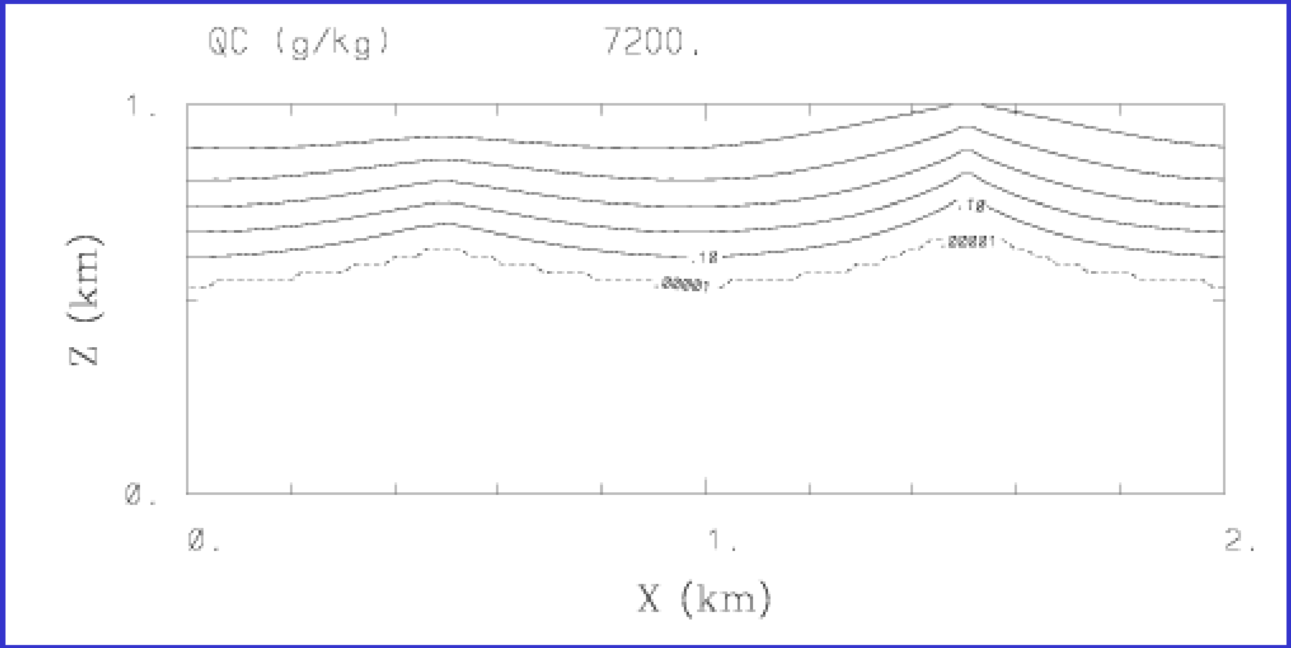
$$\frac{\partial \rho_o q_v}{\partial t} + \nabla \cdot (\rho_o \mathbf{u} q_v) = -C_d + EVAP$$

$$\frac{\partial \rho_o q_c}{\partial t} + \nabla \cdot (\rho_o \mathbf{u} q_c) = C_d - AUT - ACC$$

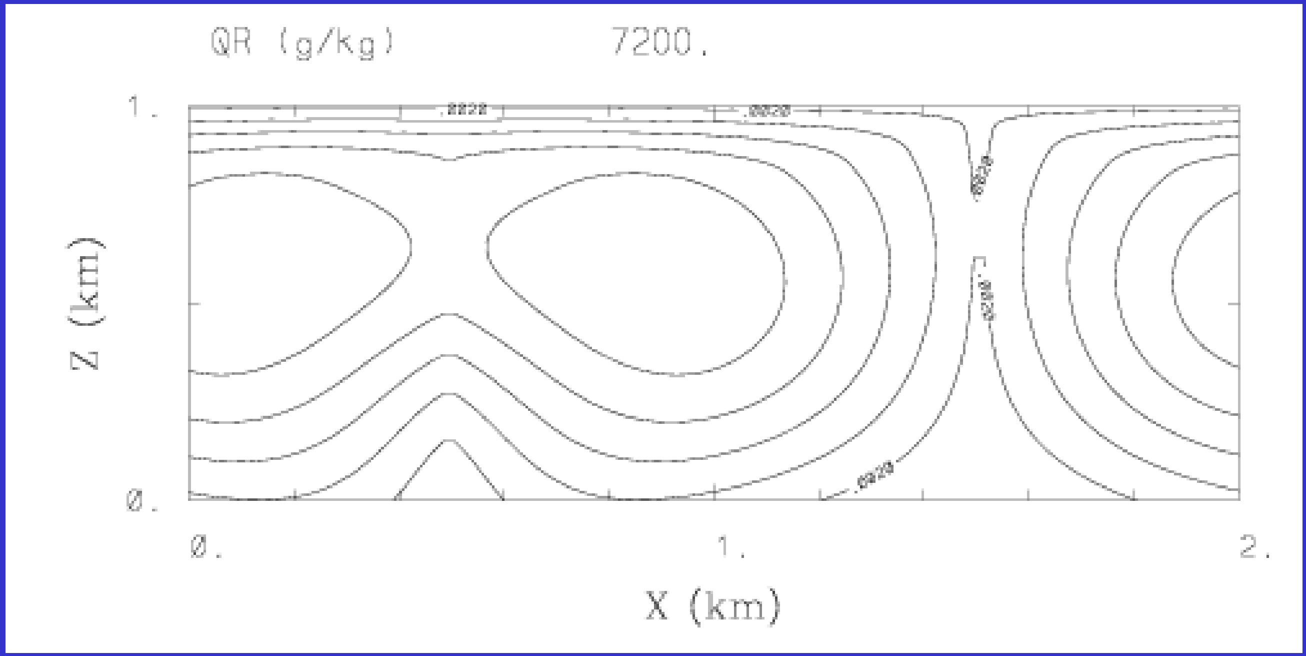
$$\frac{\partial \rho_o q_r}{\partial t} + \nabla \cdot [\rho_o (\mathbf{u} - \underline{V_T^r \mathbf{k}) q_r}] = AUT + ACC - EVAP$$

We need something more complicated than a rising parcel as rain has to fall out. One possibility is to use the *kinematic (prescribed flow) framework*...





Cloud water and rain (drizzle) fields after 2 hrs (almost quasi-equilibrium…)



Modeling of ice microphysics

Ice processes:

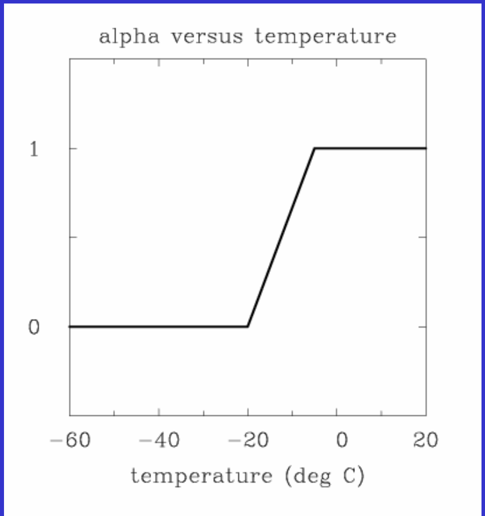
Ice initiation is the main problem: both primary (typically freezing of cloud droplets) and secondary (the ice multiplication problem).

Unlike warm-rain microphysics, where cloud droplets and rain/drizzle drops are well separated in the radius space, growth of ice phase is continuous in size/mass space. Both diffusional and accretional growth are important.

Complexity of ice crystal shapes (“habits”).

Equilibrium approach

(simple extension of a warm-rain scheme, coded in EULAG)



SIMPLE BULK ICE MODEL (Grabowski 1998):

$$\frac{d\theta}{dt} = \frac{L_v \theta}{c_p T}(COND - DIFF)$$

$$\frac{dq_v}{dt} = -COND + DIFF$$

$$\frac{dq_c}{dt} = COND - AUTC - ACCR$$

$$\frac{dq_p}{dt} = \frac{1}{\rho}\frac{\partial}{\partial z}(\rho q_p v_t) + AUTC + ACCR - DIFF$$

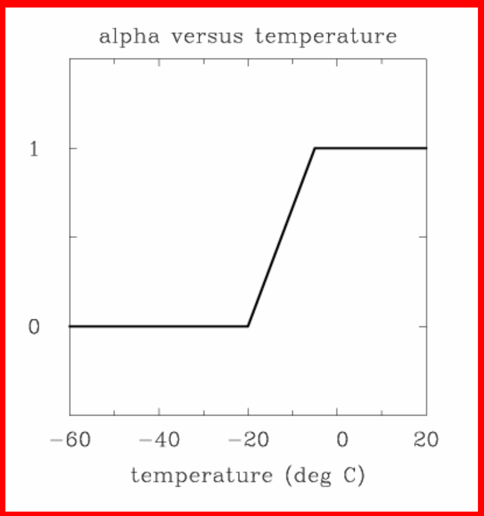
θ - potential temperature
q_v - *water vapor* mixing ratio
q_c - *cloud condensate (water or ice)* mixing ratio
q_p - *precipitation water (rain or snow)* mixing ratio
$COND$ - condensation rate (saturation adjustment)
$DIFF$ - diffusional growth rate
$AUTC$ - "autoconversion" rate: $q_c \rightarrow q_p$
$ACCR$ - accretion rate: $q_c, q_p \rightarrow q_p$

saturation: $q_{vs} = \alpha q_{vw} + (1-\alpha) q_{vi}$

cloud water: $q_w = \alpha q_c$; cloud ice: $q_i = (1-\alpha) q_c$
rain: $q_r = \alpha q_p$; snow: $q_s = (1-\alpha) q_p$
$DIFF = DIFF_r + DIFF_s$
$AUTC = AUTC_r + AUTC_s$
$ACCR = ACCR_r + ACCR_s$
$v_t = \alpha v_t(q_r) + (1-\alpha) v_t(q_s)$

Equilibrium approach

(simple extension of a warm-rain scheme, coded in EULAG)



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$$\frac{dq_c}{dt} = COND - AUTC - ACCR$$

$$\frac{dq_p}{dt} = \frac{1}{\rho}\frac{\partial}{\partial z}(\rho q_p v_t) + AUTC + ACCR - DIFF$$

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q_v - *water vapor* mixing ratio
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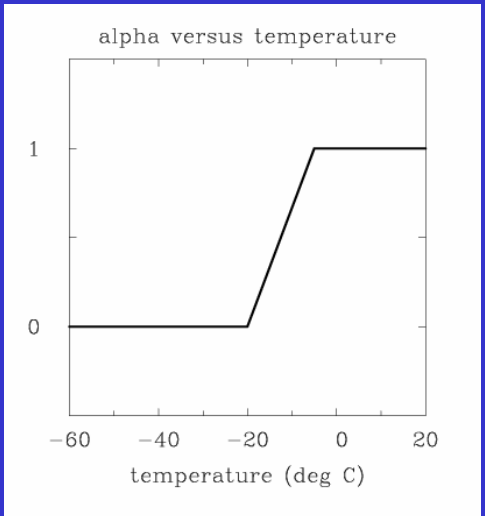
saturation: $q_{vs} = \alpha q_{vw} + (1-\alpha) q_{vi}$

cloud water: $q_w = \alpha q_c$; cloud ice: $q_i = (1-\alpha) q_c$
rain: $q_r = \alpha q_p$; snow: $q_s = (1-\alpha) q_p$
$DIFF = DIFF_r + DIFF_s$
$AUTC = AUTC_r + AUTC_s$
$ACCR = ACCR_r + ACCR_s$
$v_t = \alpha v_t(q_r) + (1-\alpha) v_t(q_s)$

Equilibrium approach

(simple extension of a warm-rain scheme, coded in EULAG)



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θ - potential temperature
q_v - *water vapor* mixing ratio
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$AUTC$ - "autoconversion" rate: $q_c \rightarrow q_p$
$ACCR$ - accretion rate: $q_c, q_p \rightarrow q_p$

saturation: $q_{vs} = \alpha q_{vw} + (1 - \alpha) q_{vi}$

cloud water: $q_w = \alpha q_c$; cloud ice: $q_i = (1 - \alpha) q_c$
rain: $q_r = \alpha q_p$; snow: $q_s = (1 - \alpha) q_p$
$DIFF = DIFF_r + DIFF_s$
$AUTC = AUTC_r + AUTC_s$
$ACCR = ACCR_r + ACCR_s$
$v_t = \alpha v_t(q_r) + (1 - \alpha) v_t(q_s)$

Non-equilibrium approach (i.e., predicting supersaturation for temperatures below 0 degC)

Lin et al. 1983

Rutledge and Hobbs 1984

BULK MODEL WITH ICE MICROPHYSICS:

- potential temperature θ:

$$\frac{d\theta}{dt} = \frac{L_v \theta_e}{c_p T_e} S_1 + \frac{L_s \theta_e}{c_p T_e} S_2 + \frac{L_f \theta_e}{c_p T_e} S_3$$

- water vapor mixing ratio q_v:

$$\frac{dq_v}{dt} = S_v$$

- cloud condensate variables q_c^i, $i = 1, N_c$ (typically, $N_c = 2$: cloud water, cloud ice):

$$\frac{dq_c^i}{dt} = S_c^i$$

- precipitating water variables q_p^i, $i = 1, N_p$: (typically, $N_p = 3$: rain, snow, graupel/hail):

$$\frac{dq_p^i}{dt} = \frac{1}{\rho} \frac{\partial}{\partial z} \left(\rho q_p^i v_t^i \right) + S_p^i$$

S – various sources/sinks due to phase changes

Traditional approach to bulk cloud microphysics

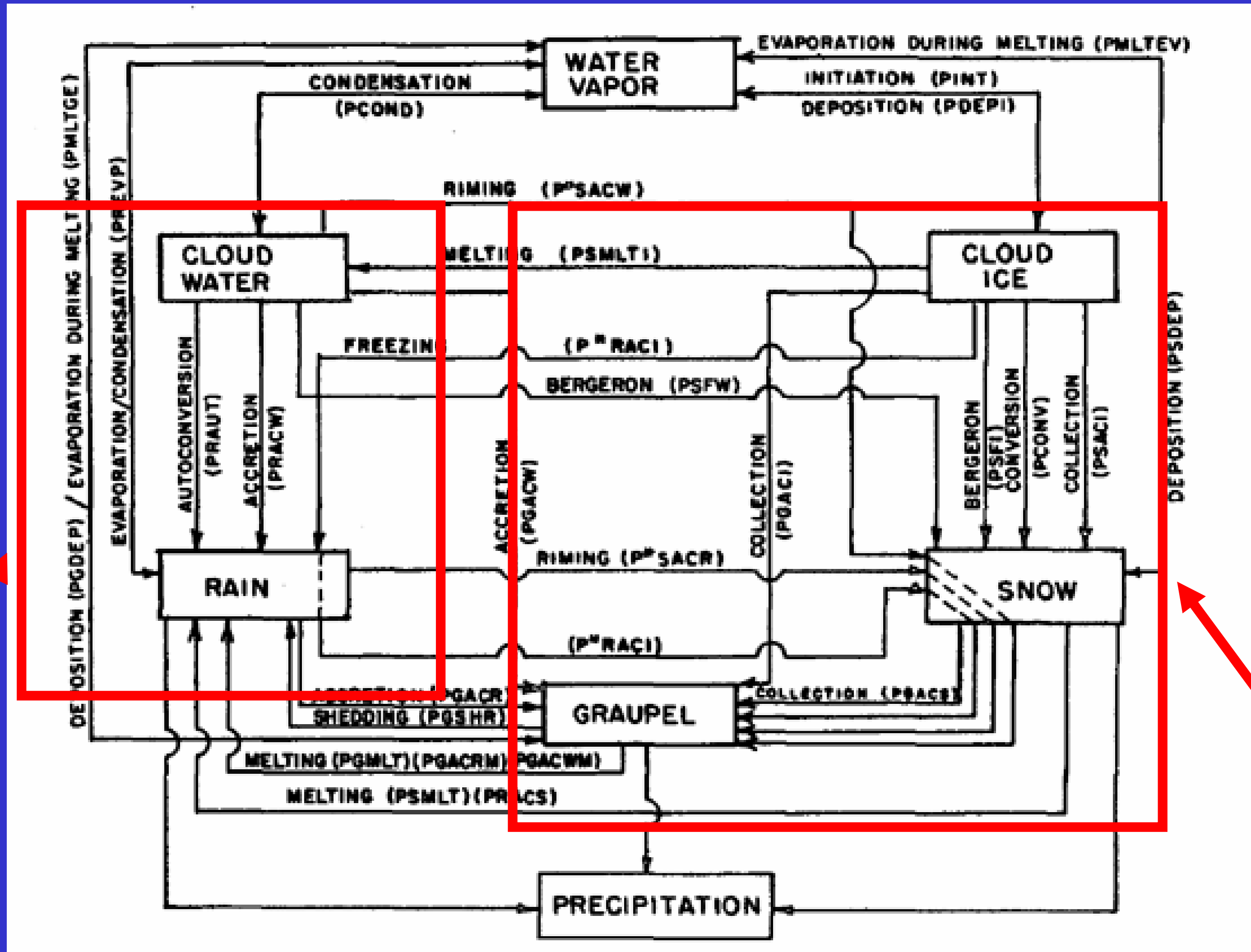


FIG. 1. Schematic depicting the cloud and precipitation processes included in the model for the study of narrow cold-frontal rainbands.

Warm-rain part

Ice part

Lin et al. JCAM 1983, Rutledge and Hobbs JAS 1984

BULK RAIN/ICE MODEL
(Lin et al. 1983, Rutledge and Hobbs 1984)

$$\frac{\partial \rho_o \theta}{\partial t} + \nabla \cdot (\rho_o \mathbf{u} \theta) = \frac{L_v \theta_e}{c_p T_e} S_1 + \frac{L_s \theta_e}{c_p T_e} S_2 + \frac{L_f \theta_e}{c_p T_e} S_3 + D_\theta$$

Water vapor

$$\frac{\partial \rho_o q_v}{\partial t} + \nabla \cdot (\rho_o \mathbf{u} q_v) = S_4 + D_{q_v}$$

Cloud water

$$\frac{\partial \rho_o q_c}{\partial t} + \nabla \cdot (\rho_o \mathbf{u} q_c) = S_5 + D_{q_c}$$

Cloud ice

$$\frac{\partial \rho_o q_i}{\partial t} + \nabla \cdot (\rho_o \mathbf{u} q_i) = S_6 + D_{q_i}$$

Rain

$$\frac{\partial \rho_o q_r}{\partial t} + \nabla \cdot [\rho_o (\mathbf{u} - V_T^r \mathbf{k}) q_r] = S_7 + D_{q_r}$$

Snow

$$\frac{\partial \rho_o q_s}{\partial t} + \nabla \cdot [\rho_o (\mathbf{u} - V_T^s \mathbf{k}) q_s] = S_8 + D_{q_s}$$

Graupel (or hail)

$$\frac{\partial \rho_o q_g}{\partial t} + \nabla \cdot [\rho_o (\mathbf{u} - V_T^g \mathbf{k}) q_g] = S_9 + D_{q_g}$$

Non-equilibrium approach coded up in EULAG based on Koenig and Murray (1976): splitting ice field into:

ice A – freezing of cloud droplets (homo and hetero), and

ice B – freezing if raindrops through interactions between ice A and rain.

$$\frac{\partial \rho_o \theta}{\partial t} + \nabla(\rho_o \boldsymbol{u} \theta) = \mathscr{F}_\theta \equiv$$

$$\frac{L_v \theta_e}{c_p T_e}(\mathrm{COND} - \mathrm{REVP}) + \frac{L_s \theta_e}{c_p T_e}(\mathrm{DEPA} + \mathrm{DEPB} + \mathrm{HOMA1})$$

$$+ \frac{L_f \theta_e}{c_p T_e}(\mathrm{RIMA} + \mathrm{RIMB} + \mathrm{HOMA2} + \mathrm{HETA} + \mathrm{HETB1} - \mathrm{MELA} - \mathrm{MELB}) \tag{1a}$$

$$\frac{\partial \rho_o q_v}{\partial t} + \nabla(\rho_o \boldsymbol{u} q_v) = \mathscr{F}_{q_v} \equiv -\mathrm{COND} + \mathrm{REVP} - \mathrm{DEPA} - \mathrm{DEPB} - \mathrm{HOMA1} \tag{1b}$$

$$\frac{\partial \rho_o q_c}{\partial t} + \nabla(\rho_o \boldsymbol{u} q_c) = \mathscr{F}_{q_c} \equiv \mathrm{COND} - \mathrm{AUTC} - \mathrm{RCOL} - \mathrm{RIMA} - \mathrm{RIMB1}$$

$$- \mathrm{HOMA2} - \mathrm{HETA} \tag{1c}$$

$$\frac{\partial \rho_o q_r}{\partial t} + \nabla[\rho_o(\boldsymbol{u} - V_r \boldsymbol{k}) q_r] = \mathscr{F}_{q_r} \equiv -\mathrm{REVP} + \mathrm{AUTC} + \mathrm{RCOL} + \mathrm{MELA}$$

$$+ \mathrm{MELB} - \mathrm{HETB1} - \mathrm{RIMB2} \tag{1d}$$

$$\frac{\partial \rho_o q_A}{\partial t} + \nabla[\rho_o(\boldsymbol{u} - V_A \boldsymbol{k}) q_A] = \mathscr{F}_{q_A} \equiv \mathrm{HOMA} + \mathrm{HETA} + \mathrm{DEPA} + \mathrm{RIMA}$$

$$- \mathrm{MELA} - \mathrm{HETB2} \tag{1e}$$

$$\frac{\partial \rho_o q_B}{\partial t} + \nabla[\rho_o(\boldsymbol{u} - V_B \boldsymbol{k}) q_B] = \mathscr{F}_{q_B} \equiv \mathrm{HETB} + \mathrm{DEPB} + \mathrm{RIMB} - \mathrm{MELB} \tag{1f}$$

- COND ($q_v \rightarrow q_c$): condensation of water vapor to form cloud water;
- AUTC ($q_c \rightarrow q_r$): autoconversion of cloud water into rain (initiation of the rain field);
- RCOL ($q_c \rightarrow q_r$): collection of cloud water by rain water;
- REVP ($q_r \rightarrow q_v$): evaporation of rain;
- HETA ($q_c \rightarrow q_A$): heterogeneous nucleation of ice A (freezing of cloud droplets);
- HOMA = HOMA1 + HOMA2 ($q_v,\ q_c \rightarrow q_A$): homogeneous nucleation of ice A;
- HETB = HETB1 + HETB2 ($q_r,\ q_A \rightarrow q_B$): nucleation of ice B due to interaction of rain with ice A;
- DEPA ($q_v \rightarrow q_A$): growth of ice A due to deposition of water vapor;
- DEPB ($q_v \rightarrow q_B$): growth of ice B due to deposition of water vapor;
- RIMA ($q_c \rightarrow q_A$): growth of ice A due to accretion of cloud water (i.e., growth by riming);
- RIMB = RIMB1 + RIMB2 ($q_c,\ q_r \rightarrow q_B$): growth of ice B by accretion of cloud water and rain;
- MELA ($q_A \rightarrow q_r$): melting of ice A to form rain; and
- MELB ($q_B \rightarrow q_r$): melting of ice B to form rain.

II. Bin microphysics models

(only for warm rain)

BIN-RESOLVING WARM MICROPHYSICS:

CONDENSATION

Introducing *spectral density function* $\phi(r,t)$:

$$\phi(r,t) \equiv \frac{d\,N(r,t)}{d\,r}$$

$d\,N(r,t)$ is the concentration (per unit mass as mixing ratio) of droplets in radius interval $(r,\ r{+}dr)$.

Continuity equation for growth by condensation:

$$\frac{\partial\phi(r,t)}{\partial t} + \frac{\partial}{\partial r}\left(\frac{d\,r}{d\,t}\ \phi(r,t)\right) = 0$$

where $\frac{d\,r}{d\,t}$ is growth rate of a droplet with radius r:

$$\frac{d\,r}{d\,t} = \frac{A(T,p)\ S}{r}$$

$S = \frac{q_v}{q_{vs}} - 1$ is the supersaturation; q_v is the ambient water vapor mixing ratio; $q_{vs}(p,T)$ is the saturated water vapor mixing ratio.

BIN-RESOLVING WARM MICROPHYSICS:

NUCLEATION AND CONDENSATION

Continuity equation for nucleation and growth by condensation:

$$\frac{\partial \phi(r,t)}{\partial t} + \frac{\partial}{\partial r}\left(\frac{d\,r}{d\,t}\ \phi(r,t)\right) = S_{nucl}$$

where S_{nucl} is the source associated with nucleation of cloud droplets (CCN activation).

S_{nucl} adds droplets (typically to the 1st bin) when the concentration of droplets $\int \phi\, dr$ is smaller than the total concentration for a given supersaturation S, the latter given by aS^b.

Activation of CCN:

N - total concentration of activated droplets

S – supersaturation

$$N = a\,S^b$$

a, b – parameters characterizing CCN

0 < b < 1 (typically, b=0.5)

a~100 cm^{-3} maritime/clean

a~1,000 cm^{-3} continental/polluted

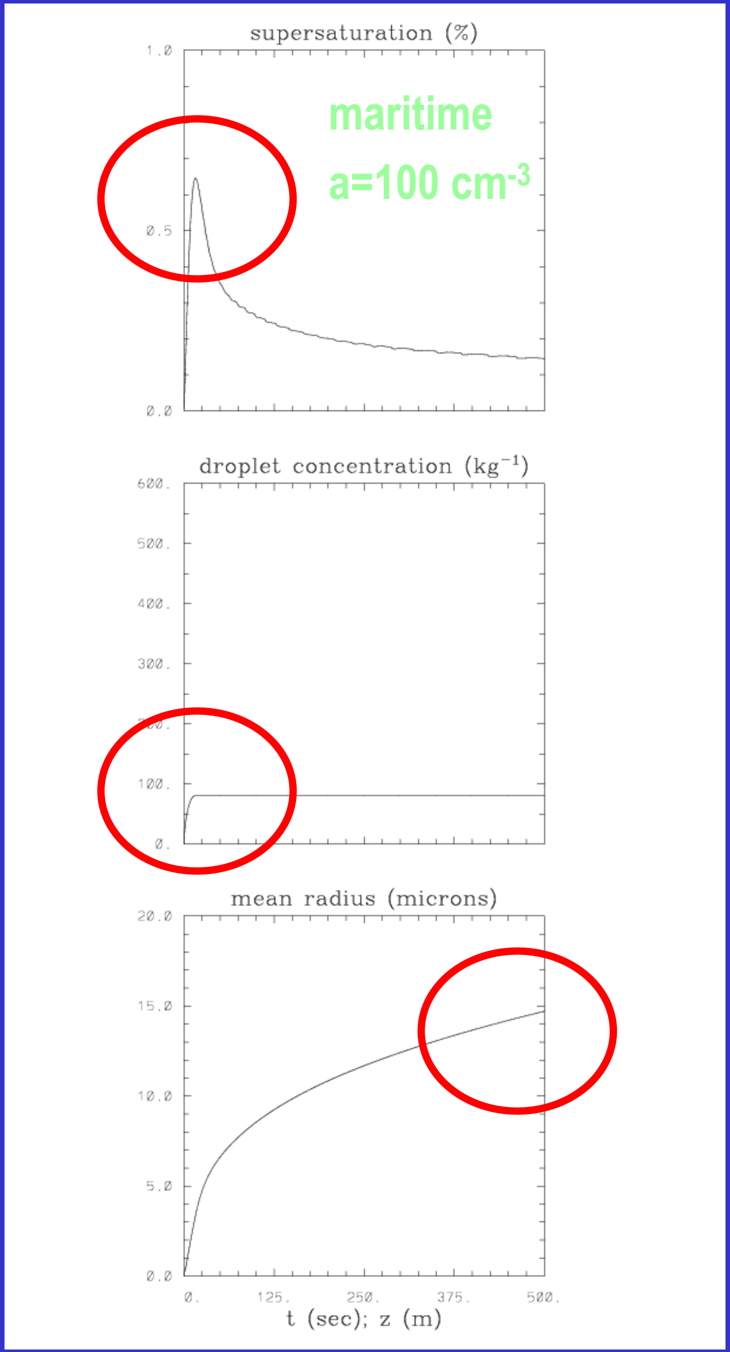
Computational example:

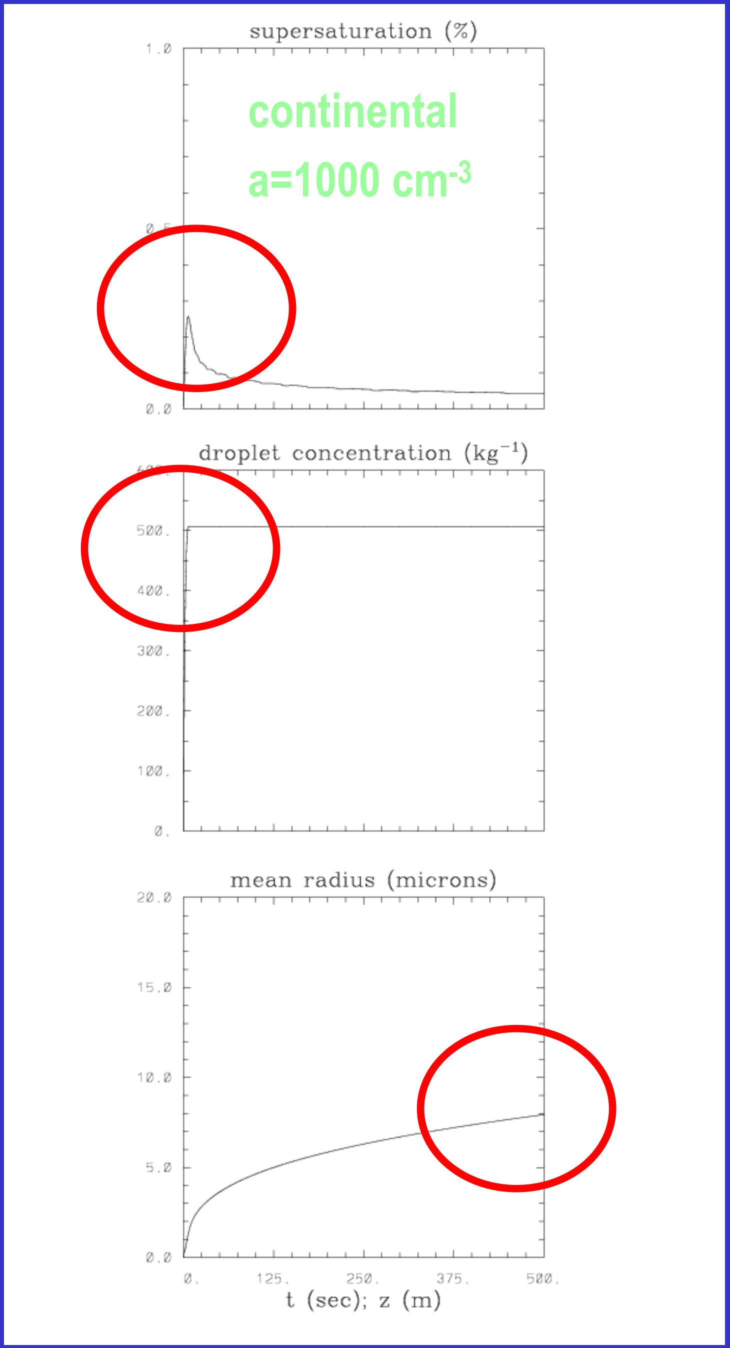
Nucleation and growth of cloud droplets in a parcel of air rising with vertical velocity of 1 m/s;

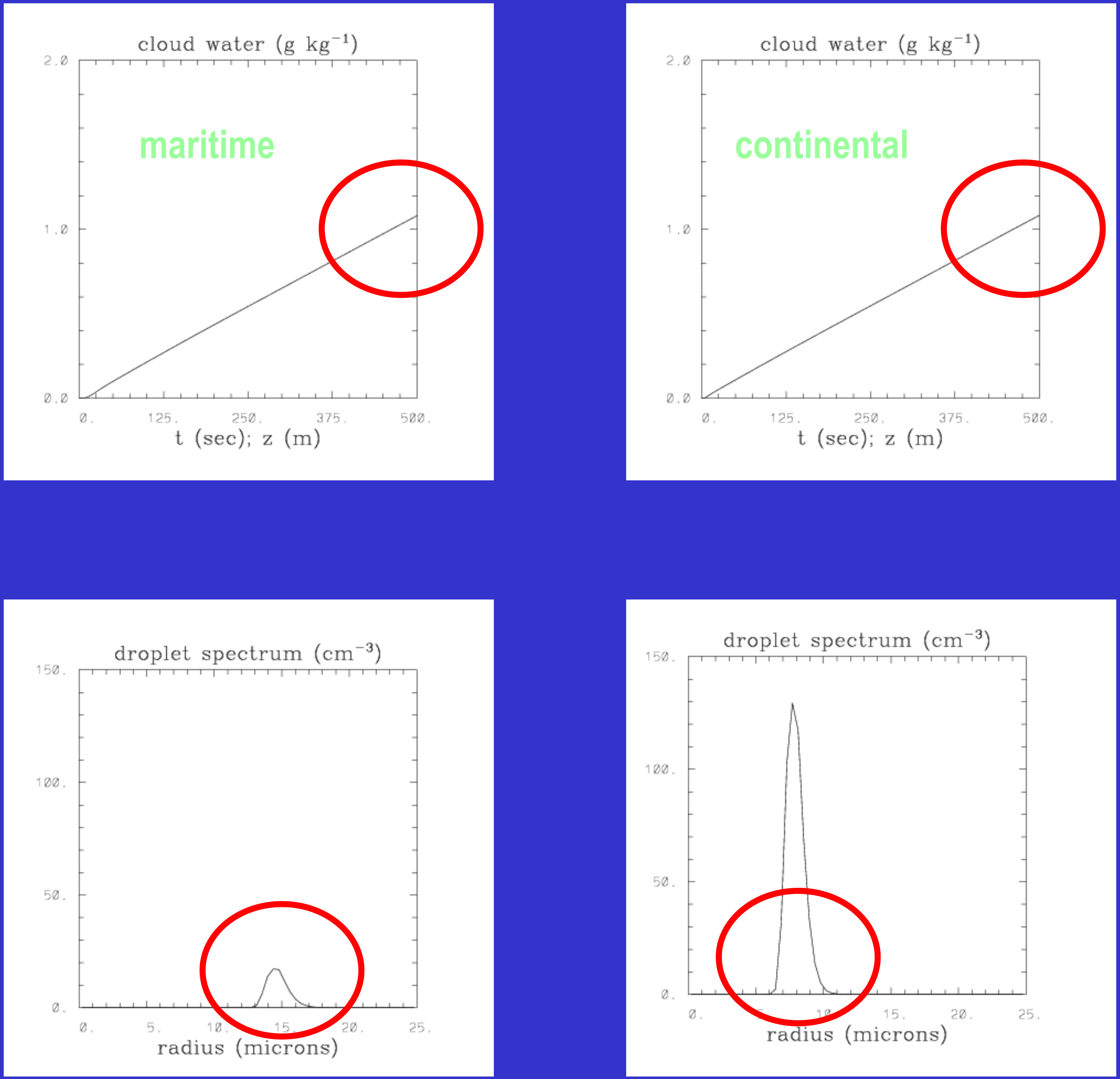
60 bins used;

1D flux-form advection applied in the radius space;

Difference between continental/polluted and maritime/pristine aerosols


supersaturation (%)
maritime
a=100 cm-3
droplet concentration (kg-1)
mean radius (microns)
t (sec); z (m)


supersaturation (%)
continental
a=1000 cm-3
droplet concentration (kg-1)
mean radius (microns)
t (sec); z (m)



cloud water (g kg^{-1})
maritime
2.0
1.0
0.0
0.
125.
250.
375.
500.
t (sec); z (m)
cloud water (g kg^{-1})
continental
2.0
1.0
0.0
0.
125.
250.
375.
500.
t (sec); z (m)
droplet spectrum (cm^{-3})
150.
100.
50.
0.
0.
5.
10.
15.
20.
25.
radius (microns)
droplet spectrum (cm^{-3})
150.
100.
50.
0.
0.
15.
20.
25.
radius (microns)

Simulations of cloud-clear air interfacial mixing in decaying moist turbulence setup (final stages of cloud entrainment)

Andrejczuk et al., JAS, 2004; 2006; 2008-submitted

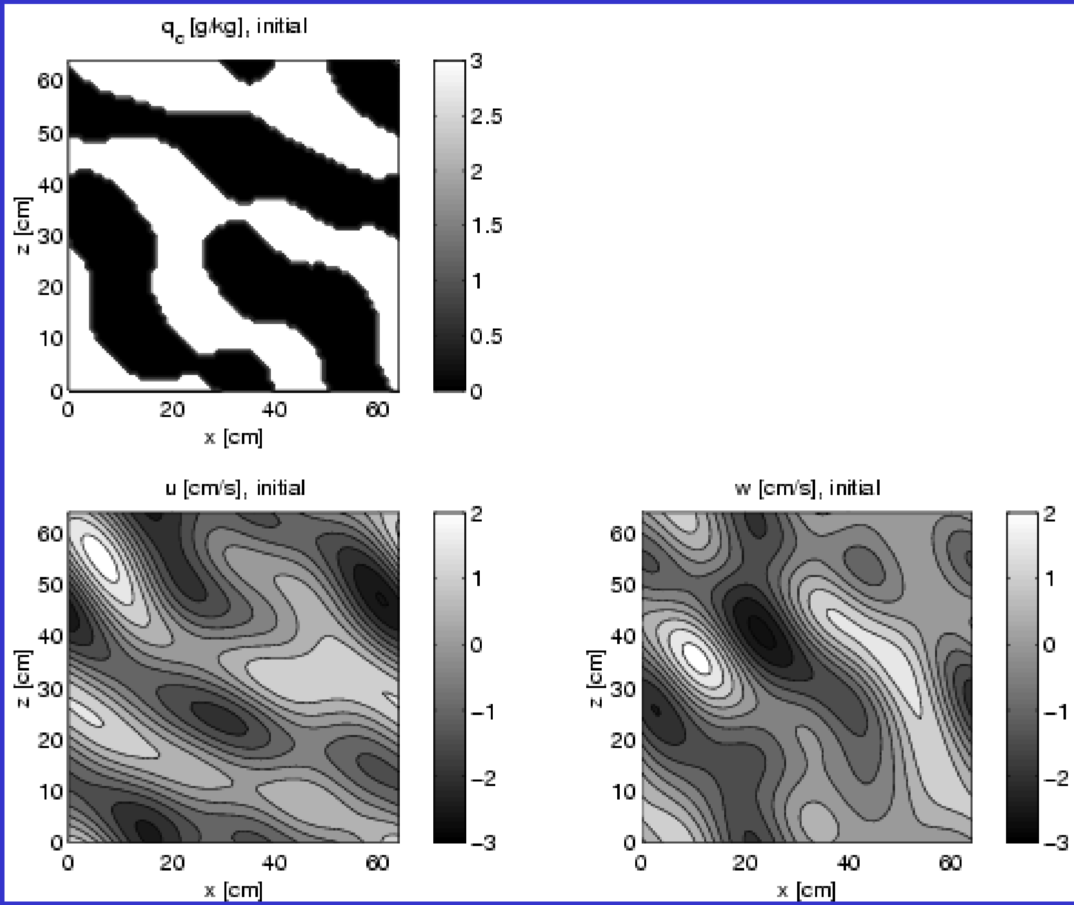
T=293 K
(both cloud and clear air)

RH=65%

q_c=3.2 g/kg

(filaments neutrally buoyant)

16 bins for cloud droplets



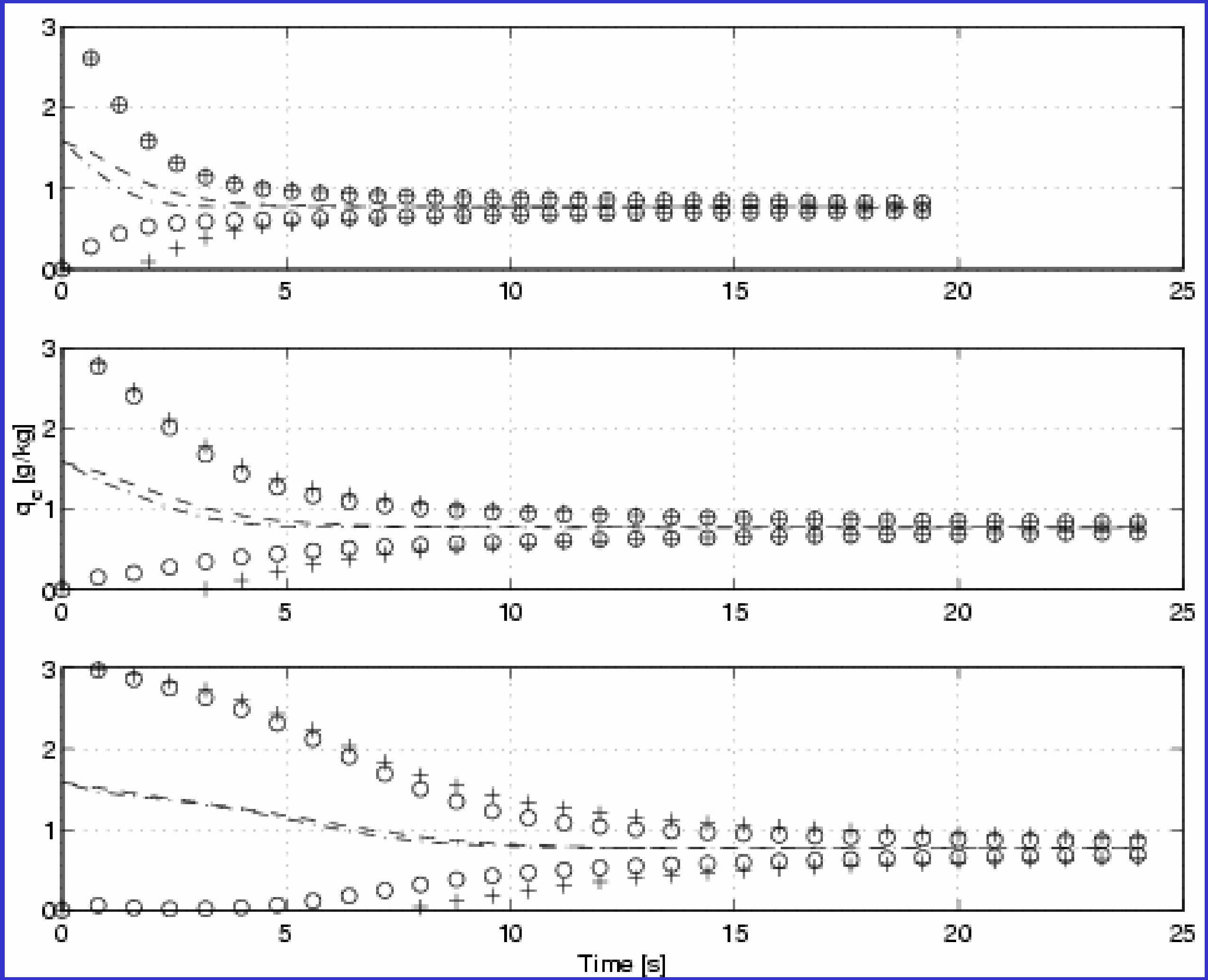
Velocity scale for low TKE (x10 for high TKE)

Evolution of the cloud water

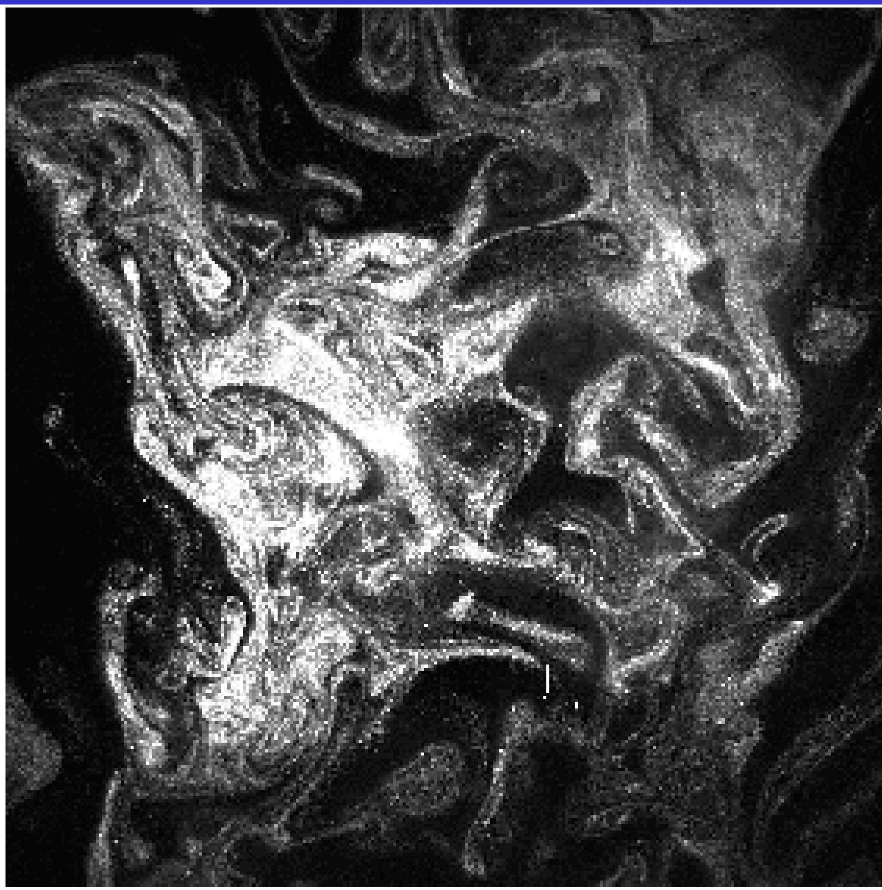
High TKE

Moderate TKE

Low TKE



Laboratory experiment

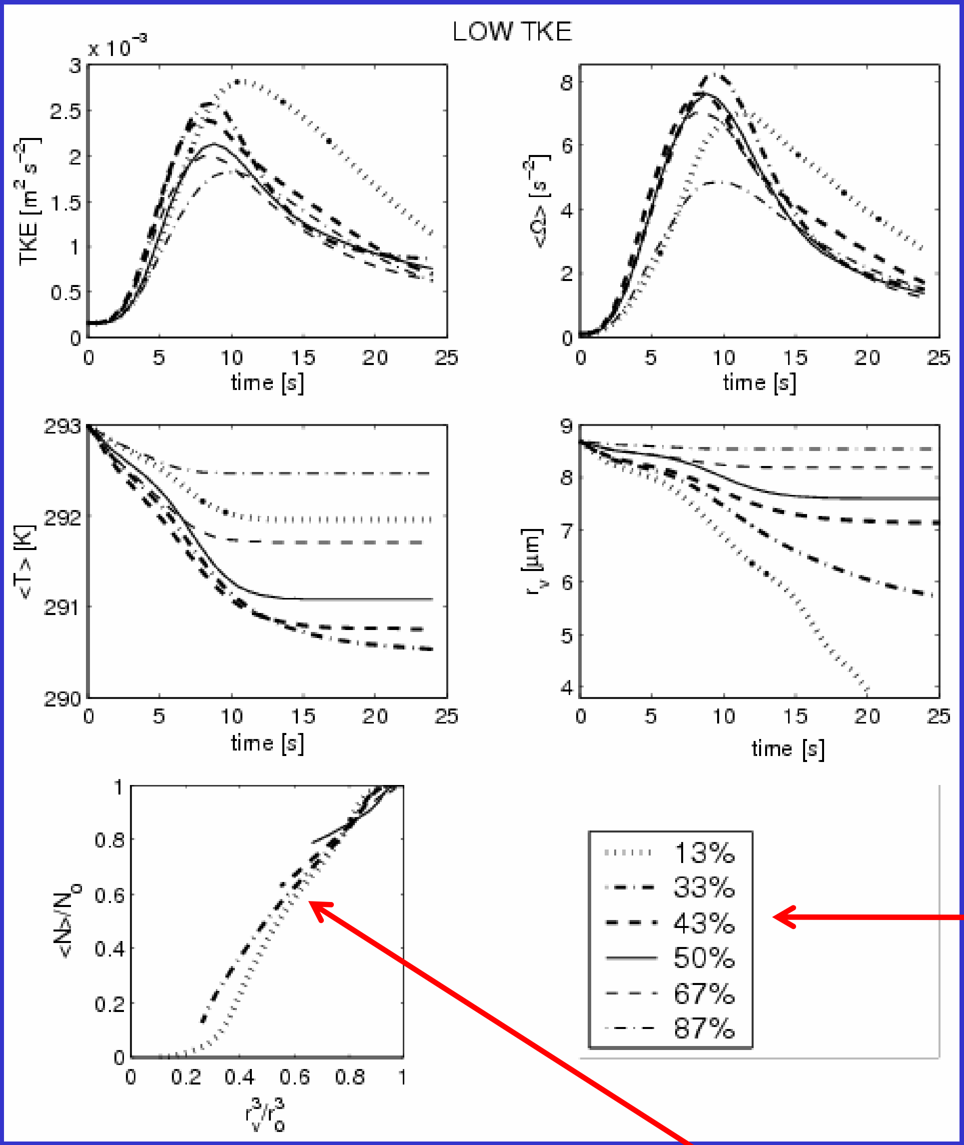


60 cm

Model simulation

60 cm

Andrejczuk et al JAS 2006



The percentage represents the initial volume fraction of cloudy air.

Evolution of the number of droplets N and their mean volume radius r_v, both normalized by the initial values

Adding coalescence (code ready to use, but there is an issue with droplet nucleation, see Grabowski and Wang, ACPD 2008)

BIN-RESOLVING WARM RAIN MODEL:

$$\frac{d\theta}{dt} = \frac{L_v \theta}{c_p T} \sum_{i=1}^{N} C_d^{(i)}$$

$$\frac{dq_v}{dt} = -\sum_{i=1}^{N} C_d^{(i)}$$

for $i = 1, N$:

$$\frac{dq_c^{(i)}}{dt} = \frac{1}{\rho} \frac{\partial}{\partial z} \left[\rho q_c^{(i)} v_t(r^{(i)})\right] + \sum_{i=1}^{N} C_d^{(i)} + F_+^{(i)} - F_-^{(i)}$$

θ - potential temperature
q_v - *water vapor* mixing ratio
$q_c^{(i)}$ - *cloud water* mixing ratio for drops in size bin i ($i = 1.N$, $N \sim 100$)

$C_d^{(i)}$ - condensation/evaporation rate for drops in size bin i; depends on super/undersaturation $S = q_v/q_{vs} - 1$ and drop size $r^{(i)}$.

$F_+^{(i)}$ - source due to collisions between j and k resulting in drops in i
$F_-^{(i)}$ - sink due to collisions between i and all other bins

- Bulk warm rain microphysics is a relatively straightforward and computationally efficient approach (e.g., just 2 variables for the warm rain);

- Problems begin for shallow clouds when microphysical details decide whether precipitation develops or not (e.g., stratocumulus, shallow convection);

- When coupled to the radiative transfer, information about cloud droplet size is needed; bulk warm rain model is not able to provide this;

- Detailed (bin-resolved) microphysics solves the above two problems, but it is very expensive (~100 extra variables) and still leaves some issues (activation);

- A reasonable compromise is to predict both the mass and the number of particles (thus, 4 variables used for the condensed water); “double-moment bulk microphysics schemes”.

III. Double-moment bulk models

Traditional approach to bulk cloud microphysics

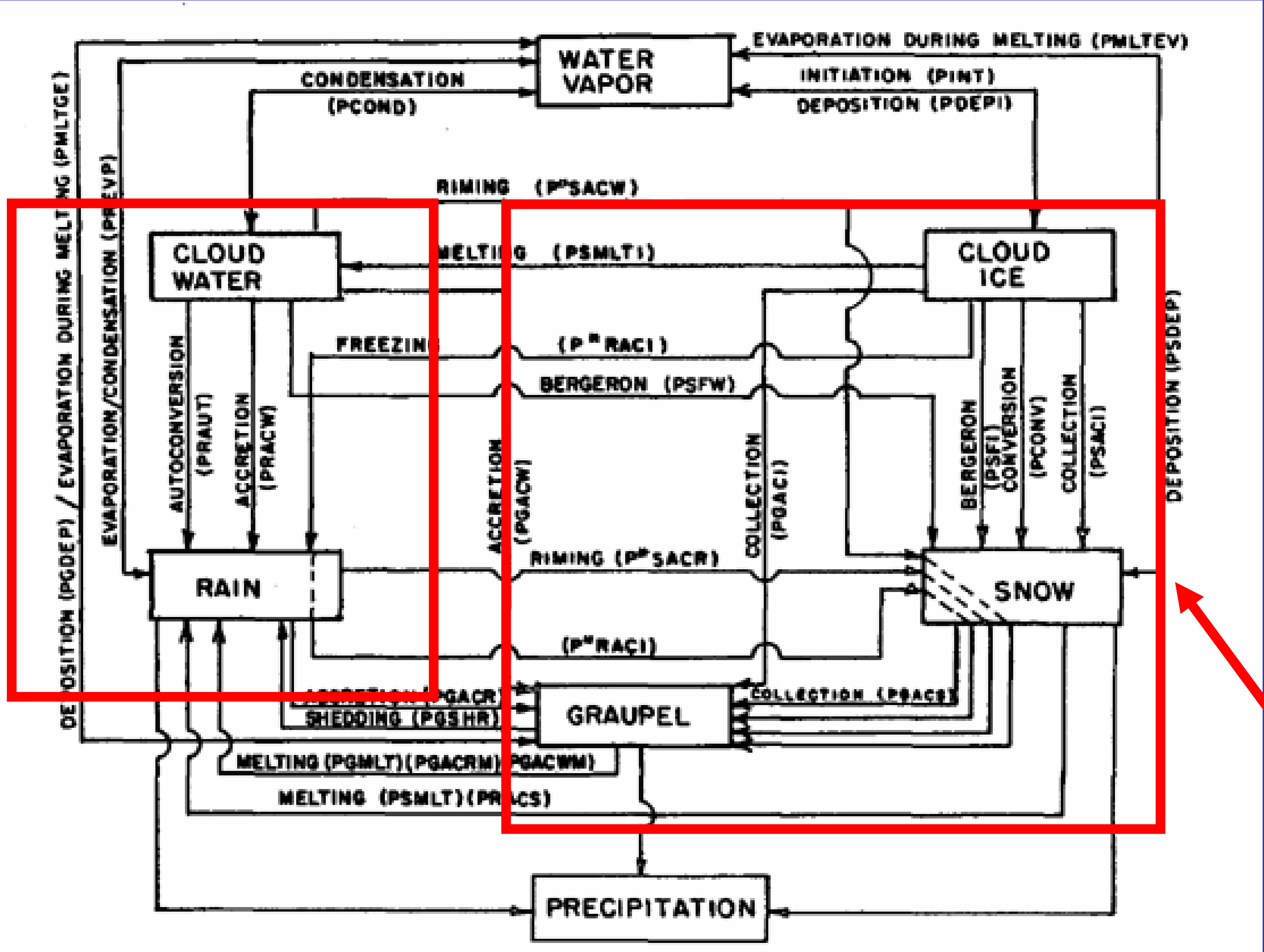


FIG. 1. Schematic depicting the cloud and precipitation processes included in the model for the study of narrow cold-frontal rainbands.

Warm-rain part

Ice part

Lin et al. JCAM 1983, Rutledge and Hobbs JAS 1984

In the original approach, only mass of cloud condensate and precipitation is predicted.

For the clouds-in-climate problem, however, prediction of particle sizes is needed (e.g., for radiative transfer). This can be accomplished by predicting mass and number of precipitation particles, i.e., the double-moment bulk scheme. This also allows including more physically-based representations of water and ice nucleation (CCN, IFN).

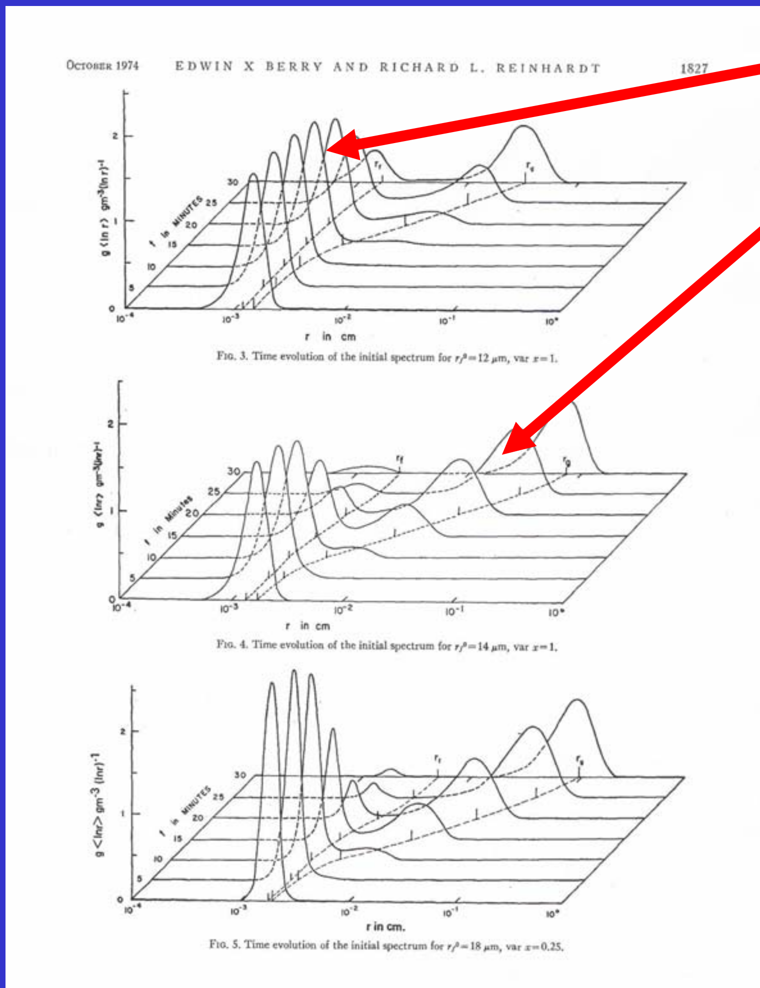
Ziegler 1985, Cohard and Pinty 2000, Khairoutdinov and Kogan 2000, Siefert and Beheng 2001, Morrison et al. 2005

Lets look at warm-rain and ice physics separately…

WARM-RAIN PHYSICS:

cloud water: q_c , N_c

drizzle/rain water: q_r , N_r



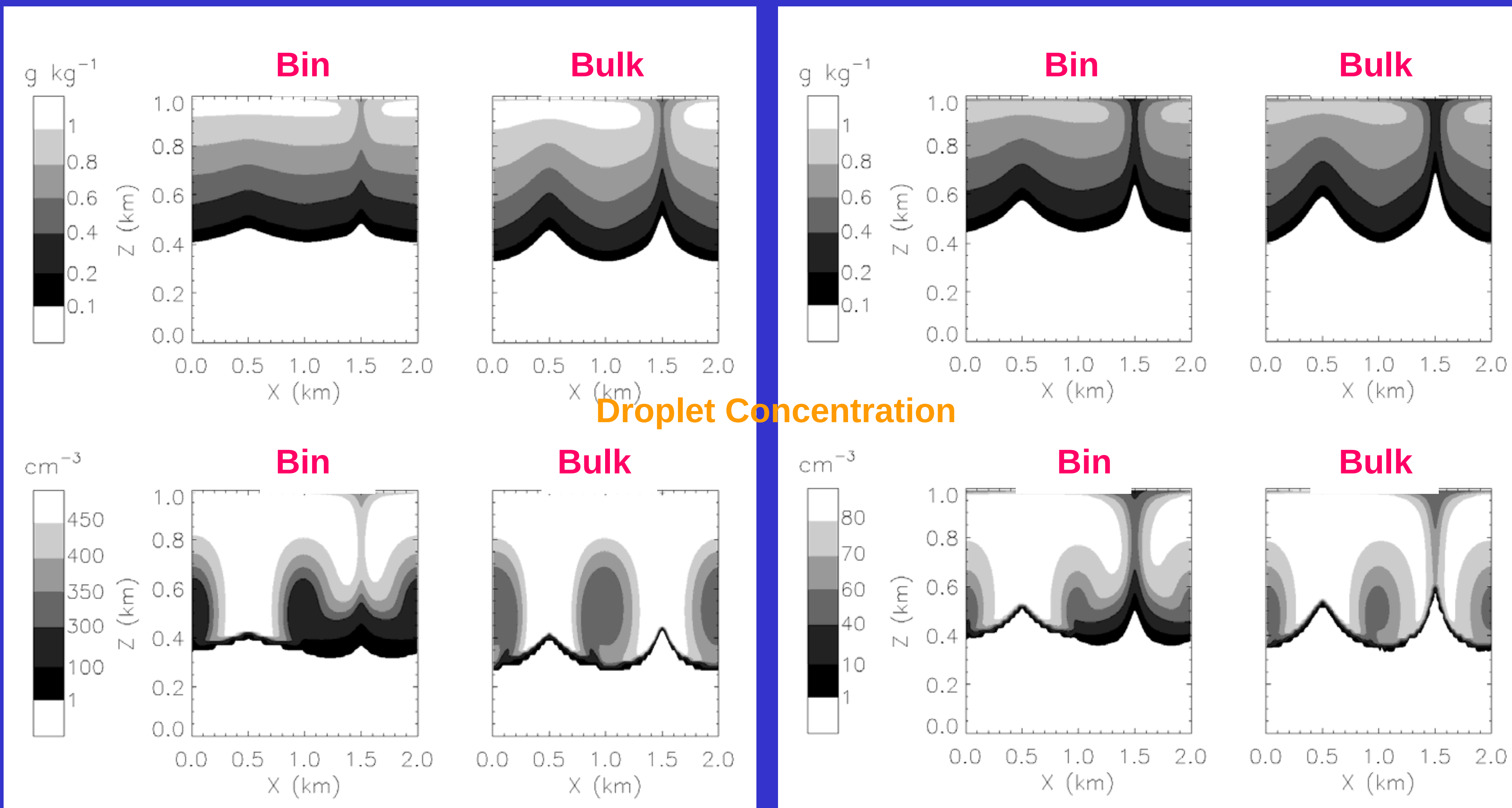
Nucleation of cloud droplets: link to CCN characteristics

Drizzle/rain development: link to mean droplet size

Results from a kinematic model of drizzling Stratocumulus

(Morrison and Grabowski JAS 2007)

POLLUTED   Droplet Mixing Ratio   PRISTINE



Microphysical transformations during sub-grid mixing

- Flexibility to treat any mixing scenario from homogeneous to extremely inhomogeneous.

$$N_f = N_i \left(\frac{q_f}{q_i}\right)^{\alpha}$$

N – droplet concentration

q – cloud water mixing ratio

- $\alpha = 1$: extremely inhomogeneous
- $\alpha = 0$: homogeneous

N_i q_i – initial (i.e., after SGS mixing)

N_f q_f – final (i.e., after SGS mixing and microphysical adjustment)

Work is underway to locally predict α…

TRADITIONAL ICE PHYSICS:

cloud ice: q_i , N_i

snow: q_s , N_s

graupel / hail: q_g , N_g

TRADITIONAL ICE PHYSICS:

cloud ice: q_i , N_i

snow: q_s , N_s

graupel / hail: q_g , N_g

Is such an approach justified?

Not really!

The ice scheme should produce various types of ice (cloud ice, snow, graupel) just by the physics of particle growth. Partitioning ice particles a priori into separate categories introduces unphysical “conversion rates” and may involve “thresholding behavior” (i.e., model solutions diverge depending whether the threshold is reached or not).

NEW ICE PHYSICS:

A double-moment three-variable ice scheme:

$$\frac{\partial N}{\partial t} + \frac{1}{\rho_a}\nabla \cdot [\rho_a(\mathbf{u} - V_N\mathbf{k})N] = \mathcal{F}_N$$

Number concentration of ice crystals, N

$$\frac{\partial q_{dep}}{\partial t} + \frac{1}{\rho_a}\nabla \cdot [\rho_a(\mathbf{u} - V_q\mathbf{k})q_{dep}] = \mathcal{F}_{q_{dep}}$$

Mixing ratio of ice mass grown by diffusion of water vapor, q_{dep}

$$\frac{\partial q_{rim}}{\partial t} + \frac{1}{\rho_a}\nabla \cdot [\rho_a(\mathbf{u} - V_q\mathbf{k})q_{rim}] = \mathcal{F}_{q_{rim}}$$

Mixing ratio of ice mass grown by riming (accretion of liquid water), q_{rim}

Morrison and Grabowski JAS 2008

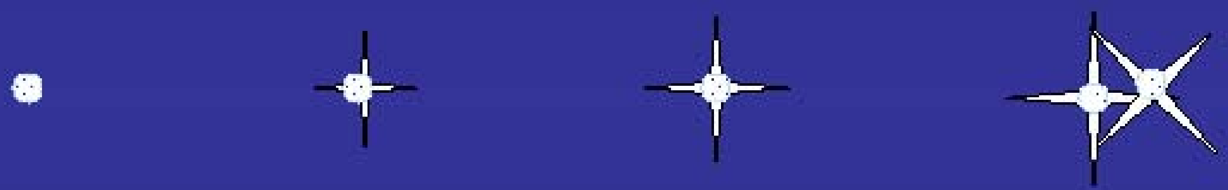
Ice particle mass-dimension (m-D) and projected-area-dimension (A-D) relationships are based on observed characteristics of ice crystals, aggregates, and graupel particles (from aircraft and ground-based observations).

$$m = \alpha D^{\beta}$$

$$A = \sigma D^{\gamma}$$

Morrison and Grabowski JAS 2008

Case without riming



- Growth by vapor deposition

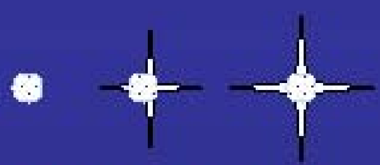
- Growth by aggregation
- Growth by vapor deposition

Smooth transition from small spherical ice to larger nonspherical crystals to aggregates.

Case with riming

Stage 1: Unrimed crystal

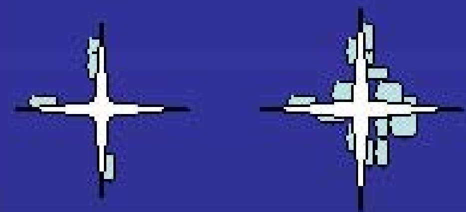
- Particle dimension and mass determined by vapor deposition



- Vapor depositional growth

Stage 2: Partially-rimed crystal

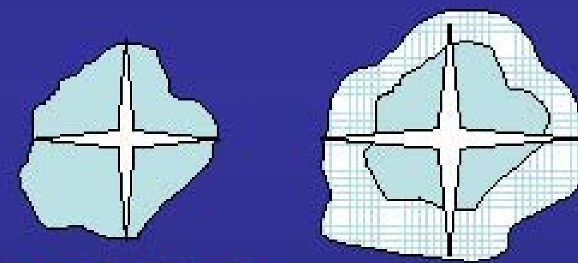
- Particle dimension determined by vapor deposition
- Mass determined by vapor deposition and riming



- Riming of crystal interstices
- Vapor depositional growth

Stage 3: Graupel

- Particle dimension determined by vapor deposition and riming
- Mass determined by vapor deposition and riming



- Complete filling-in of interstices with rime
- Further growth by riming and vapor deposition

Prediction of both riming and vapor deposition mass allows for realistic particle evolution during growth.

$$N(D) = N_o D^{\mu} e^{-\lambda D}$$

Ice particles assumed to follow gamma distribution (3 parameters: N_o, $\boldsymbol{\mu}$, λ)

$$N = \int_0^{\infty} N(D)\, dD$$

$$q_{dep} + q_{rim} \equiv q = \int_0^{\infty} m(D)\, N(D)\, dD$$

$$\frac{\partial N}{\partial t} + \frac{1}{\rho_a}\nabla \cdot [\rho_a(\mathbf{u} - V_N \mathbf{k})N] = \mathcal{F}_N$$

$$\frac{\partial q_{dep}}{\partial t} + \frac{1}{\rho_a}\nabla \cdot [\rho_a(\mathbf{u} - V_q \mathbf{k})q_{dep}] = \mathcal{F}_{q_{dep}}$$

$$\frac{\partial q_{rim}}{\partial t} + \frac{1}{\rho_a}\nabla \cdot [\rho_a(\mathbf{u} - V_q \mathbf{k})q_{rim}] = \mathcal{F}_{q_{rim}}$$

$$\mu = 0.076\lambda^{0.8} - 2\ ; \quad 0 \le \mu \le 6$$

Diagnostic relationship based on cloud observations (Heymsfield 2003)

Morrison and Grabowski JAS 2008

rimed mass fraction F_r:

$$F_r = \frac{q_{rim}}{q_{dep} + q_{rim}} \approx \frac{m_{rim}}{m_{dep} + m_{rim}}$$

assumed constant across the spectrum of ice particles

$F_r = 0$ — ice particle grown by diffusion/aggregation

$F_r \rightarrow 1$ — graupel (or hail)

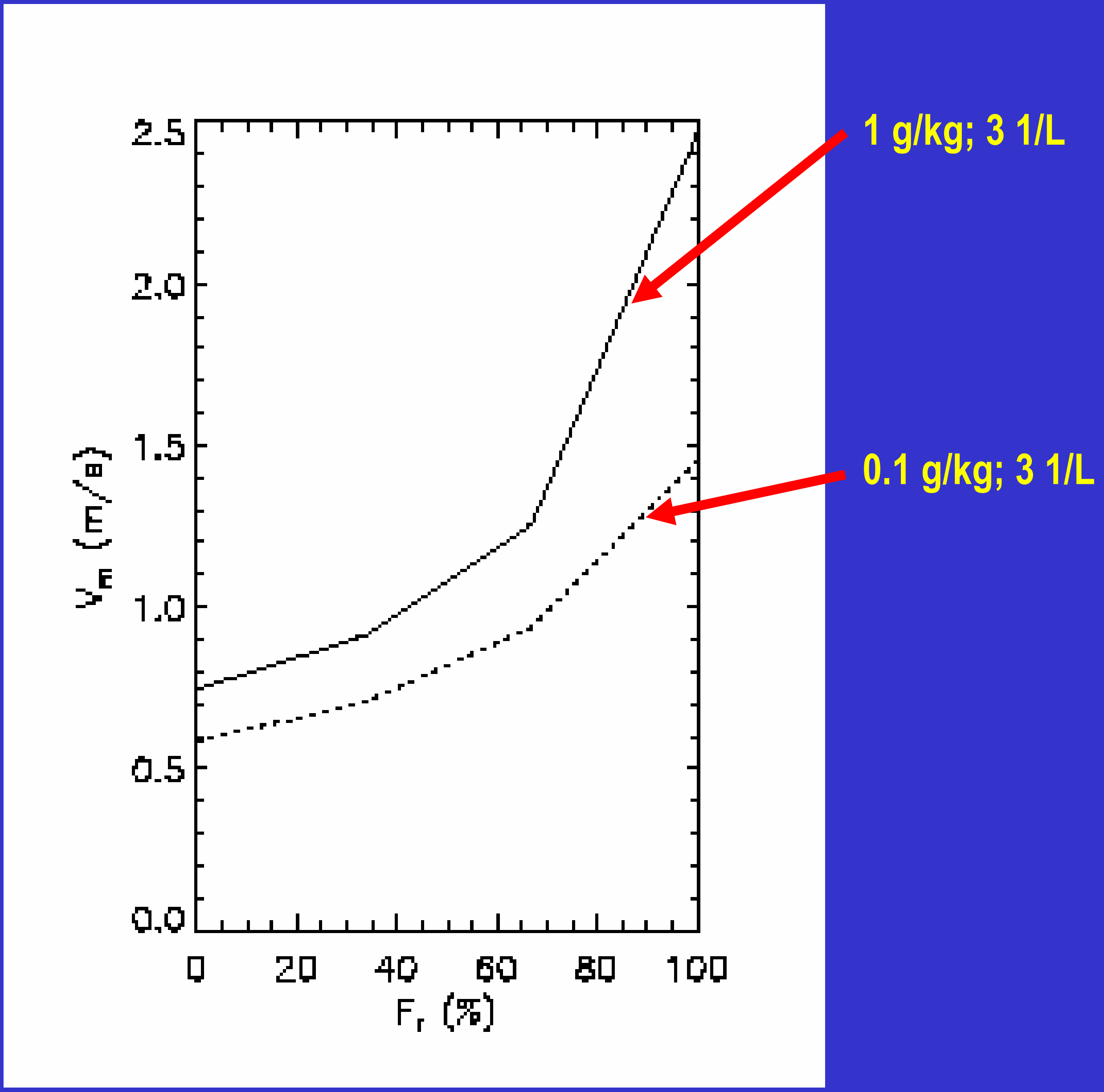
$0 < F_r < 1$ — rimed ice particle:

size D given by the mass grown by diffusion $m_{dep} = (1 - F_r)m$;

$m = m_{rim} + m_{dep}$ *is the total mass of ice particle*

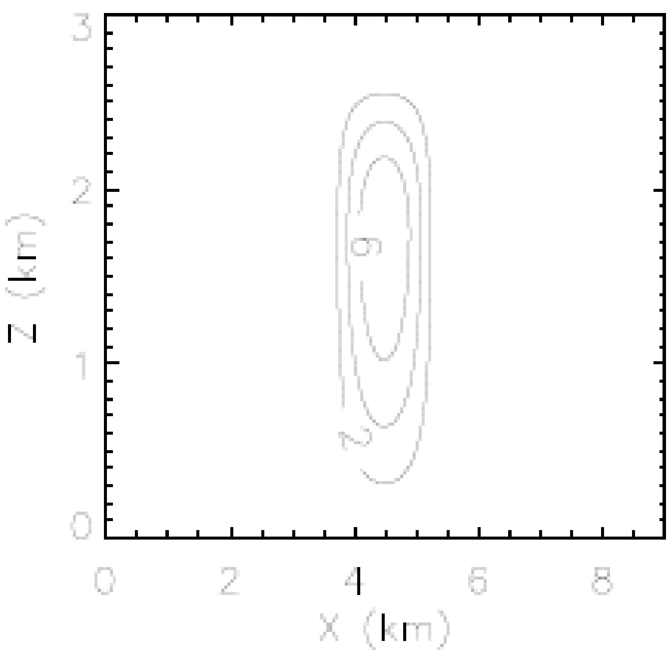
Morrison and Grabowski JAS 2008

Parameterization of ice mass fallspeed. Note gradual increase with the rimed fraction F_r



Morrison and Grabowski JAS 2008

Example of the application of the new ice scheme: precipitation development in a small convective cloud: 2D (x-z) prescribed-flow framework (low-level convergence, upper-level divergence, evolving-in-time updraft, with weak vertical shear).



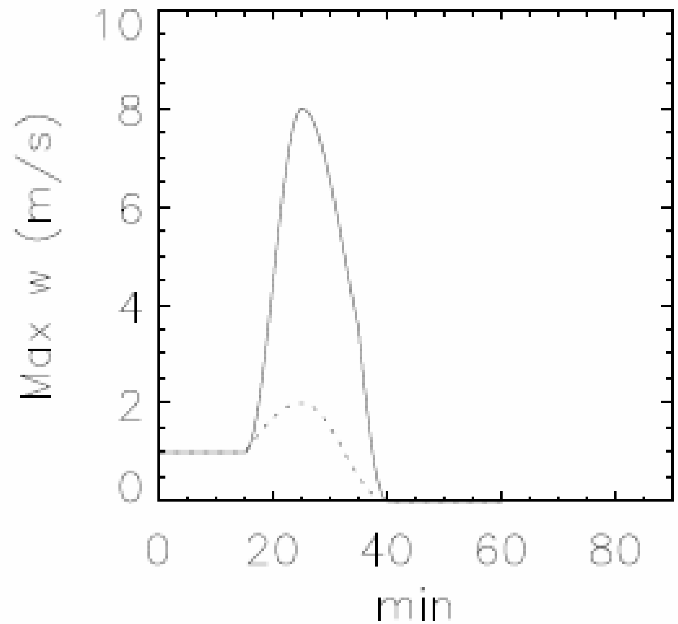
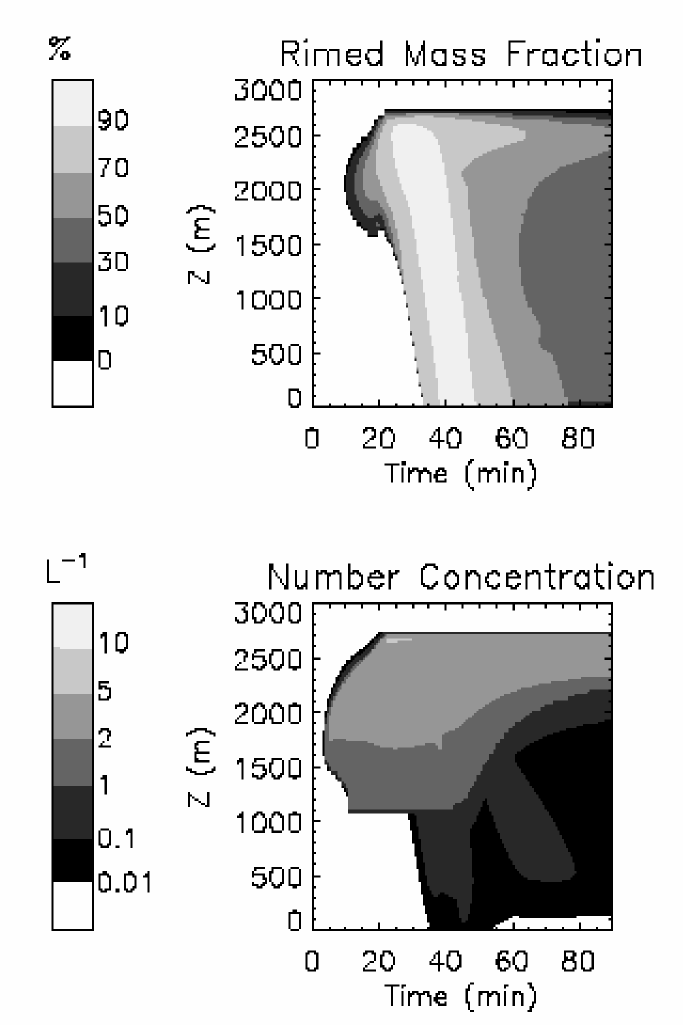
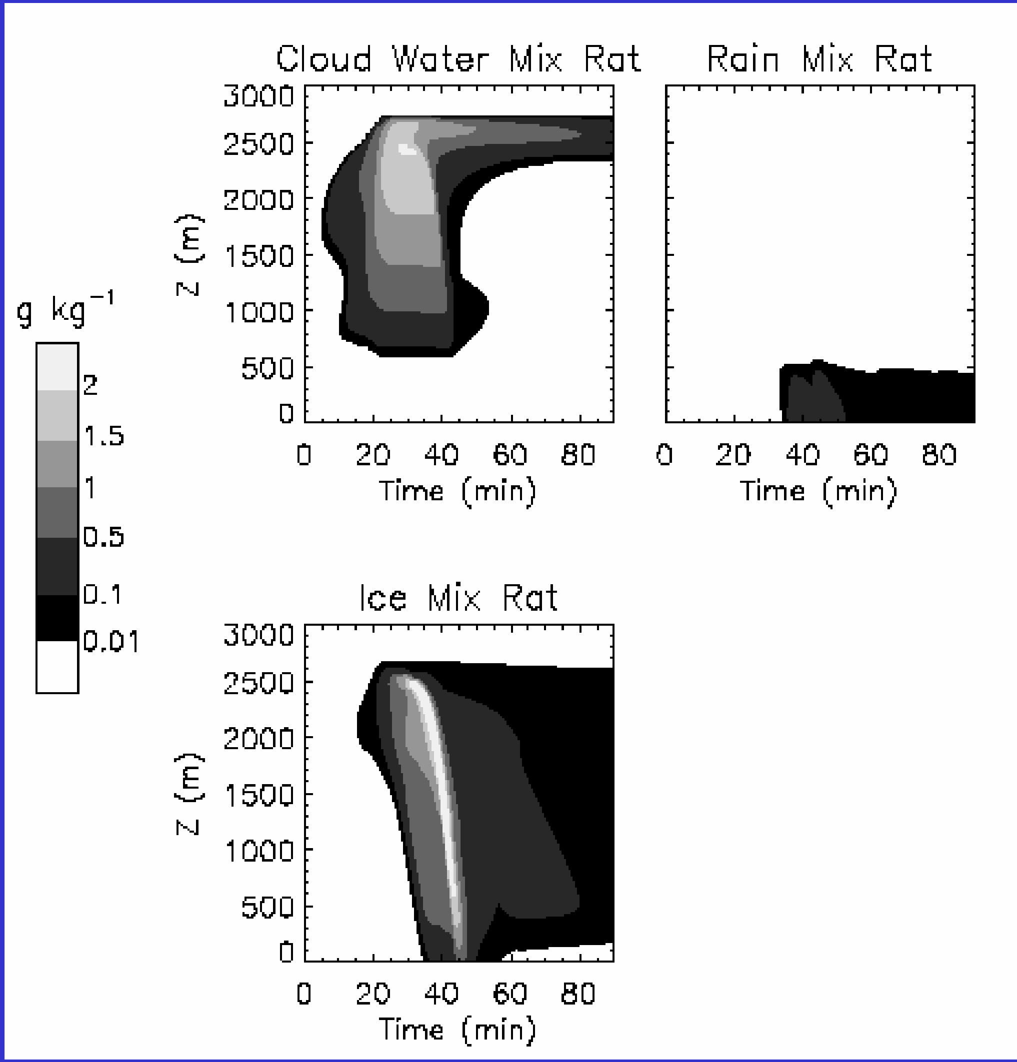


Figure 4: Maximum updraft velocity w in the X-Z plane as a function of time for peak updraft strength of 8 m/s (solid) and 2 m/s (dotted).

Morrison and Grabowski JAS 2008

Example of results:
evolutions of horizontal maximum at each level





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Concluding thoughts:

Representation of convection in large-scale models of weather and climate used to be the key issue in the past…

However, continental-scale cloud-resolving (or "convection permitting") numerical weather prediction (e.g., using WRF or COSMO models), climate modeling using super-parameterization, and development of global cloud-resolving models (e.g., at Japan's Earth Simulator) suggest that the emphasis in the future will be on the representation of cloud microphysics (and perhaps other small-scale processes, like turbulence).