# Coupling the Dynamics of Boundary Layers and Evolutionary Dunes

Great Sand Dunes Nat. Park, CO,USA (P.S. & P.O.)

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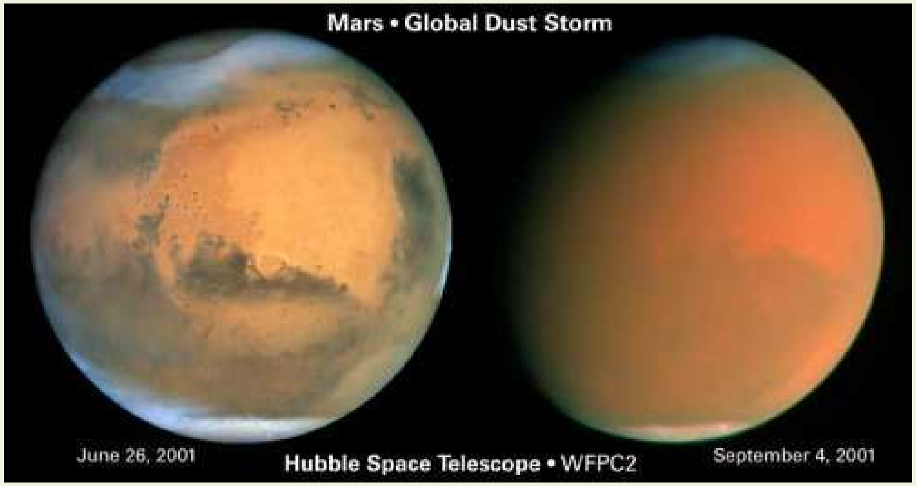
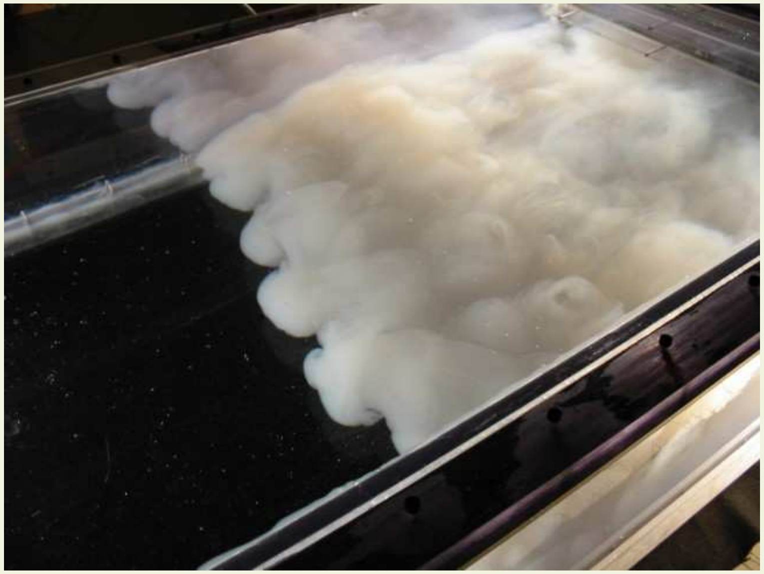
# 1. Sedimentary transport and landforms. *(a) Keys*

- **Scales of the problem**
  - Planetary scale




Mars before and after the great Martian dust storm of 2001
(From MGS, Mars Global Surveyor, and Hubble Space Telescope)

- Local scale
  - ∗ Sediment dominated currents



Turbidity Currents. Lab tank
(From `www.physics.utoronto.ca`)

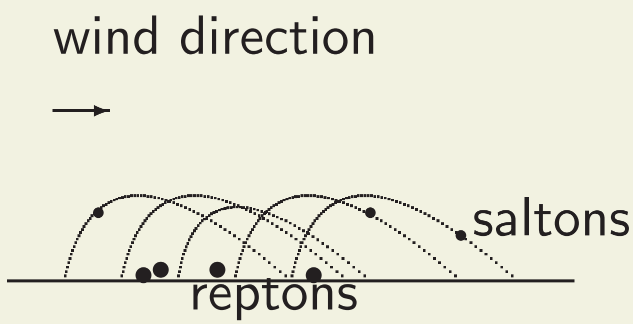
∗ Single morphologies: Barchan dunes


Simple barchan (1), Large simple barchan (2), Megabarchan (3) and ripple patterns (4). Location: 8 km. SE Chimbote, Perú. (by J. McCauley, USGS,1971)

∗ Complex morphologies: Dune fields

Great Sand Dunes Nat. Park, CO, USA
(by P. Smolarkiewicz & P. Ortiz, 2003)

– Micro-scale dynamics: Saltation, reptation and suspension

- **Turbulence**
- **Separating SPBL**
- **Active role of landform**: Intricate geometry time - dependent boundary forcing. Active in transport: slopes and avalanches

## 1. Sediment transport and landforms. *(b) Solutions*

- QSA: Quasi steady approximations
  - Extreme wind scenarios: Full coupling
- Pre requisites of the model:
  - Time dependent curvilinear coordinate transformation
  - LES, Smagorinsky type SGS model.
  - Sediment transport model:
    - ∗ Accomodated as a Convection Diffusion PDE
    - ∗ Saltation fluxes as convective fluxes
    - ∗ Sand avalanches as diffusive fluxes.(Anisotropic, inhomogeneous)

## 2.A Fluid Model

- Formulation in generalized time dependent coordinates

$$(\bar{t}, \overline{\mathbf{x}}) \equiv \big(t, \mathcal{F}(t, \mathbf{x})\big) \ ,$$

- Incompressible Boussinesq eqs, neutrally stratified flows, phys. space,

$$\nabla \cdot \big(\rho_o \mathbf{v}\big) = 0 \ ,$$

$$d\mathbf{v}/dt = -\nabla \pi' + \mathcal{D}\big(\mathcal{E}, \nabla \mathbf{v}\big) \ ,$$

$$d\mathcal{E}/dt = \mathcal{S}(\mathcal{E}) \ ,$$

- Dependence of $\overline{x},\ \overline{y},\ \overline{z},\ \bar{t}$ on $(x, y, z, t)$:

$$\bar{t} = t,\ \overline{x} = x,\ \overline{y} = y$$
$$\overline{z} := H_0 \frac{z - h(x,y,t)}{H_0(x,y,t) - h(x,y,t)}\ ,$$

$h$: lower surface elevations, physical domain,
$H_0$: vertical extend of the transformed model domain.

- Solution of the sediment motion model: $h$: solid/fluid interface profile.

## 2.B Sediment Motion

- Evolution of the interface:

$$\rho_s \frac{\partial h}{\partial t} + \nabla_H \bullet \mathbf{q} = 0 \ ,$$

$\rho_s = \rho_m(1-\lambda)$: bulk density of the sediment. $\rho_m$, $\lambda$: density of the grain material, porosity (volume of voids/total volume); $\nabla_H$: (horizontal, physical space); $\mathbf{q}$: vertically integrated sediment mass flux.

- Saltation transport:

$$\mathbf{q}_S = C\frac{\rho}{g}\mathbf{u}_* \parallel \mathbf{u}_* \parallel^2 \max\left(0, 1 - \frac{u_\tau}{\parallel \mathbf{u}_* \parallel}\right) \ ,$$

$C$ empirical coefficient depending upon the normalized grain size; $\rho$: density of the air; $g = |\mathbf{g}|$; $\mathbf{u}_* \equiv u_* \mathbf{v} \parallel \mathbf{v} \parallel^{-1}$, friction velocity $u_* = \sqrt{\rho^{-1}\tau_w}$; $\tau_w$: wall shear stress;

- $u_\tau$: threshold value of $u_*$. Dependence on the slope:

$$u_\tau = \sqrt{\frac{\sin\theta}{\tan\alpha}\cos\gamma + \sqrt{\frac{\sin^2\theta}{\tan^2\alpha}(\cos^2\gamma - 1) + \cos^2\theta}}\; u_{\tau 0}\ ,$$

$\theta$: local slope angle; $\alpha$: angle of friction; $\gamma$: angle between local wind and slope. $u_{\tau 0}$: horizontal bed.

- Avalanche transport

Local sand avalanches beneath the brink of a dune
Great Sand Dunes Nat. Park, CO, USA
(by P. Smolarkiewicz & P. Ortiz, 2003)

Diffusion fluxes, anisotropic inhomogeneous diffusion coefficient $\mathcal{K}(\mathbf{x},t)|_{\overline{z}} = 0$ depending critically on the local slope

$$\mathbf{q}_A = -\rho_s \mathcal{K} \nabla_H h \ .$$

$$\mathcal{K} := \frac{\Lambda^2}{\Upsilon} \, \frac{1 + \mathrm{sgn}(\| \nabla_H h \| - s_C)}{2} \ ,$$

$\Lambda$ and $\Upsilon$: characteristic length and time scales.

- Total flux as advection-diffusion equation

$$\frac{\partial h}{\partial t} + \nabla_H \bullet \mathbf{U} h = \nabla_H \bullet \mathcal{K} \nabla_H h; \quad \mathbf{U} := \frac{\mathbf{q}_S}{\rho_s h} \ .$$

$\mathbf{U}$: average over potentially mobilized sand layer.

# 3. Numerical model

- Eulerian conservation law

$$\frac{\partial \rho^* \psi}{\partial \bar{t}} + \overline{\nabla} \bullet \left( \overline{\mathbf{V}}^* \psi \right) = \rho^* R \ ,$$

$\psi$: components of $\mathbf{v}$ or $\mathcal{E}$, $\overline{\mathbf{V}}^* \equiv \rho^* \overline{\mathbf{v}}^*$, $R$: rhs, $\rho^* \equiv \rho_o \overline{G}$, $\overline{G}$: Jacobian of the transformation.

- NFT algorithm (second-order accuracy)

$$\psi_{\mathbf{i}}^{n+1} = \frac{\rho^{*n}}{\rho^{*n+1}} \mathcal{A}_{\mathbf{i}} \big( \widetilde{\psi},\ \overline{\mathbf{V}}^{*n+1/2},\ \delta t \big) + 0.5 \delta t R_{\mathbf{i}}^{n+1} \ ;$$

$\psi_{\mathbf{i}}^{n+1}$: solution at $(\bar{t}^{n+1}, \overline{\mathbf{x}}_{\mathbf{i}})$. $\widetilde{\psi} \equiv \psi^n + 0.5\delta t R^n$.
For $\mathcal{A}$: fully second-order-accurate multidim. MPDATA advection scheme (PS & LM 98, ...).

- SGS forcings in $R$ explicit.

- Solution for velocity and pressure (compact form):

$$\mathbf{v_i} = \widehat{\mathbf{v}}_{\mathbf{i}} - 0.5\delta t \left( \widetilde{\mathbf{G}} \overline{\nabla} \pi' \right)_{\mathbf{i}} \ ,$$

$$\left[ \frac{\delta t}{\rho^*} \overline{\nabla} \bullet \rho^* \widetilde{\mathbf{G}}^T \left( \widehat{\mathbf{v}} - \widetilde{\mathbf{G}} \overline{\nabla} \pi'' \right) \right]_{\mathbf{i}} = 0 \ ,$$

$\widetilde{\mathbf{G}}^T \left( \widehat{\mathbf{v}} - \widetilde{\mathbf{G}} \overline{\nabla} \pi'' \right) \equiv \overline{\mathbf{v}}^s$ (J.P. & P.S.,2003).
Boundary value problem: Preconditioned GCR($k$) algorithm.

- Updated pressure, updated solenoidal velocity: updated physical and contravariant velocity components using transformations.

- Sediment Transport numerical model:
  - Explicit integration to $\mathcal{O}(\delta t^2)$ using the NFT

$$h_{\mathbf{j}}^{n+2} = \frac{\overline{G_{xy}}^{n}}{\overline{G_{xy}}^{n+2}} \mathcal{A}_{\mathbf{j}}^{H}\left(\widetilde{h},\ \overline{\mathbf{U}}^{*n},\ 2\delta t\right)\ ;$$

    $\mathcal{A}_{\mathbf{j}}^{H}$: nonoscillatory horizontal-advection operator.
    $\widetilde{h} \equiv h^n + 2\delta t \mathcal{L}\big(\mathcal{K}^n,\ h^n\big)$, $\mathcal{L}$: Laplacian
    for all $\mathbf{j} = (i, j)$.
  - Preventing negative $h$ by limiting advective fluxes.

# 4. Results

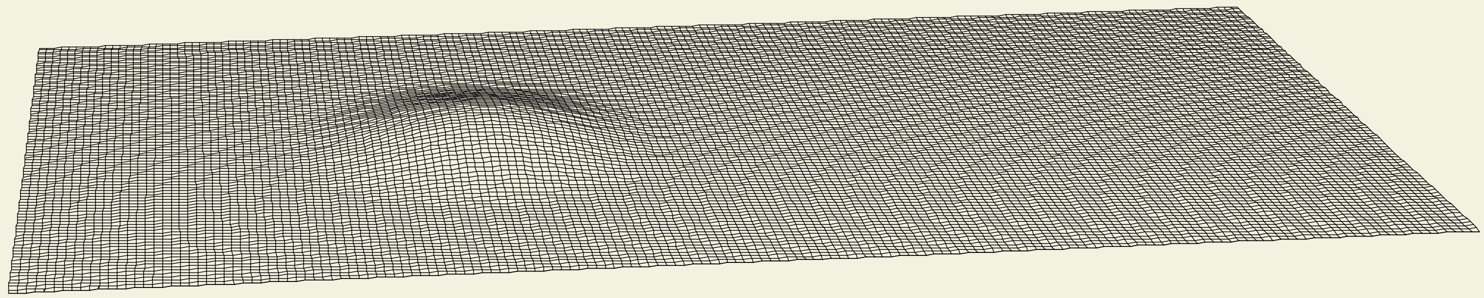
- Neutrally stratified, nonrotating Boussinesq atmosphere. Uniform ambient wind $\mathbf{v}_e = (11,\ 0,\ 0)\ \mathrm{ms}^{-1}$

- Cartesian model domain $L_x \times L_y \times L_z = 46h_o \times 32h_o \times 5.3h_o$. (for $h_o = 7.5$ m, $340 \times 180 \times 40\ \mathrm{m}^3$).

- Lower boundary at $t = 0$: cosine sandpile of height $h_o$ (range $(0.5 - 7.5)$ m. Half-width $a$, $h_o/a \approx 0.15$, centered at $(x_o, y_o) = (L_x/3, L_y/2)$, $a$ range: $\approx(3 - 50)$ m.

$$h(\mathbf{x},\ t = 0) = \begin{cases} h_o \cos^2\left(\frac{\pi r}{2a}\right) + h_b & \text{if } r/a \leq 1, \\ h_b & \text{if } r/a > 1. \end{cases}$$

# Initial conditions. Profile



$r \equiv \sqrt{(x - x_o)^2 + (y - y_o)^2}$. $h_b$: thickness of the sand layer: 0 or 5 m.

- Time scales: PBL flows $\mathcal{O}(10^3)$ s. Dune evolution $\mathcal{O}(10^6)$ s: Rescaling $C$: 1440 (minutes per day).
- Upper boundary: rigid lid. Lateral: Open and periodic (streamwise and spanwise).
- Initial condition: potential flow.
- Regular mesh (in the transformed space) $171 \times 91 \times 41$ and $333 \times 119 \times 41$.

- Surface drag coefficient in the Smagorinsky-type turbulence subgrid-scale model: $C_D = 0.01$.

- Spatial/temporal scales $\mathbf{q}_A$: $\Lambda = 0.25 \min(\delta x, \delta y)$, $\Upsilon = \delta t$.

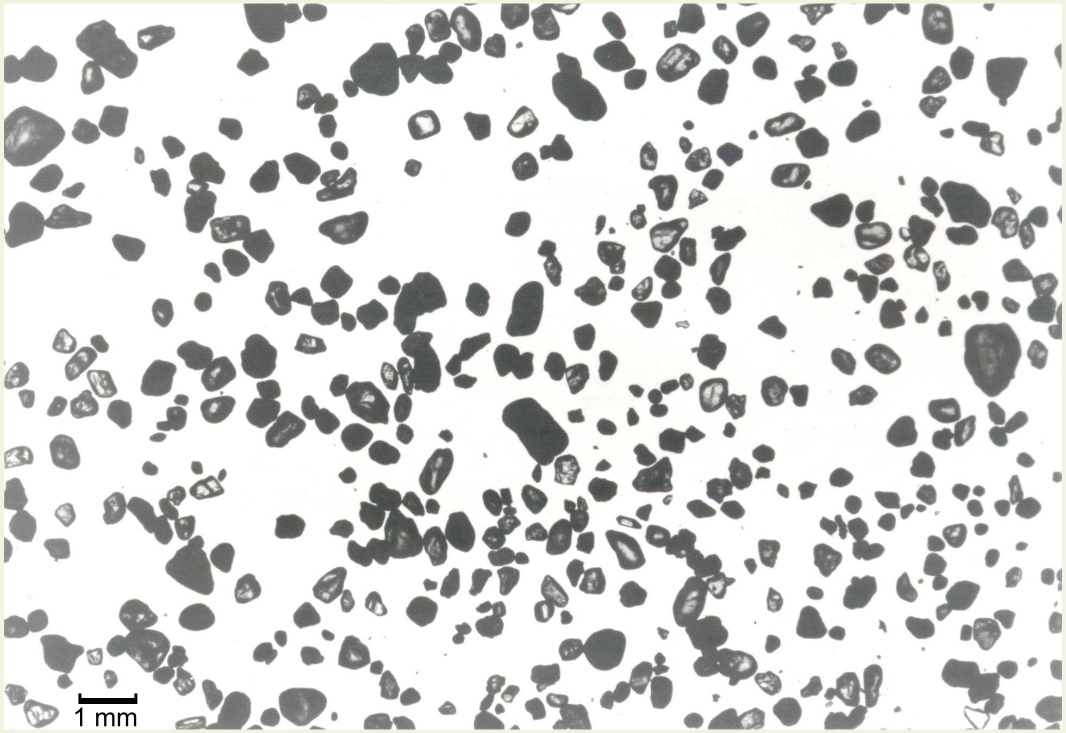
- Friction velocity:

$$\mathbf{u}_* = \kappa \frac{(\mathbf{v} - \mathbf{v} \bullet \mathbf{n})|_{z_\Delta}}{\ln(z_\Delta / z_0)}$$

  $z_\Delta$: surface-adjacent level. $z_0$: equivalent roughness length (flow-to-grains momentum transfer) $z_o = 0.001$ m. $\kappa = 0.41$ von Karman constant.

- Sediment transport parameters: $\rho_m = 2650$ kg/m$^3$ (quartz). $\lambda = 0.5$. $u_{\tau 0} = 0.22$ ms$^{-1}$.

- Collected sand sample. Sand Dunes Nat. Park



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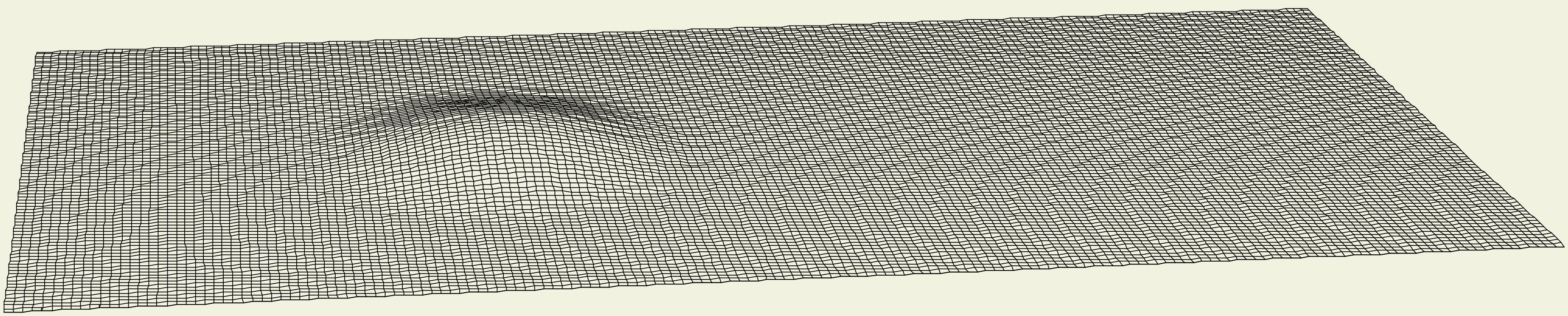
| $h_o$ [m] | $H$ [m] | $\Delta t$ [s] | $t_f/T_o$ |
|---|---|---|---|
| 7.5 | 9.5 | 0.05 | 232848 |
| 6.0 | 7.6 | 0.03 | 174636 |
| 3.0 | 3.7 | 0.02 | 232848 |
| 1.5 | 1.6 | 0.01 | 209088 |
| 1.0 | 1.0 | 0.005 | 121176 |
| 0.7 † | 0.6 | 0.002 | 105494 |
| 0.5 † | 0.4 | 0.002 | 105494 |

Experiments. $h_o$: initial heights. $H$: final max. height. $\Delta t$: time step.
$h_o/a \approx 0.15$; $h_b = 0$; $L_x/h_o \approx 46$; $L_y/h_o \approx 32$; $L_z/h_o \approx 5.3$;
$171 \times 119 \times 41$ grid points; $\Delta x \approx 0.27h_o$, $\Delta y \approx 0.27h_o$, $\Delta z \approx 0.132h_o$;
†: $h_o/a \approx 0.15$; $h_b = 0$; $L_x/h_o \approx 92$; $L_y/h_o \approx 32$; $L_z/h_o \approx 5.3$;
$333 \times 119 \times 41$ grid points; $\Delta x \approx 0.27h_o$, $\Delta y \approx 0.27h_o$, $\Delta z \approx 0.132h_o$.

# Non erodible substrate profile



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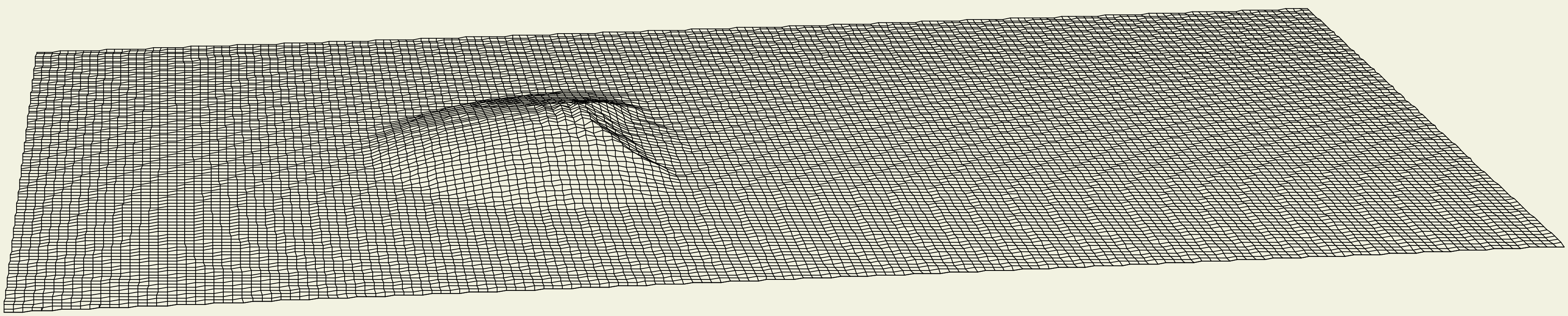

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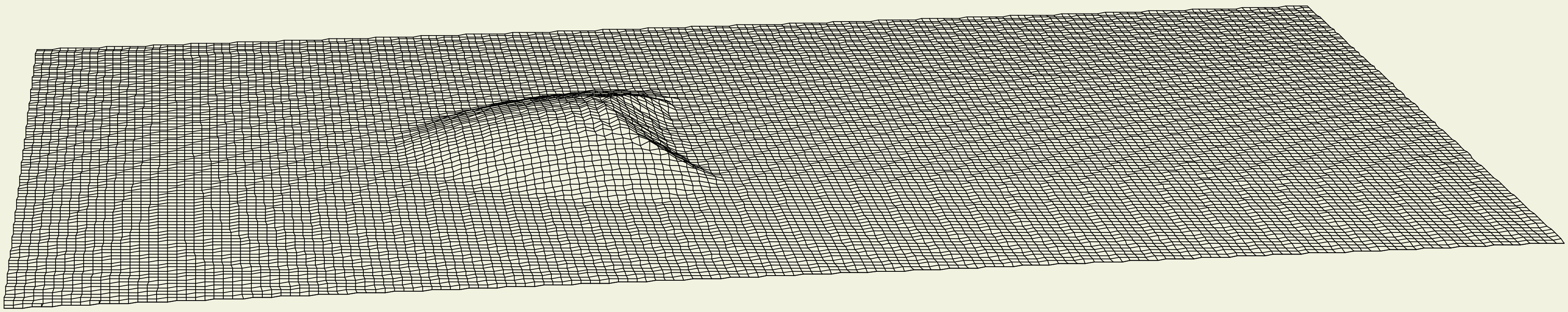

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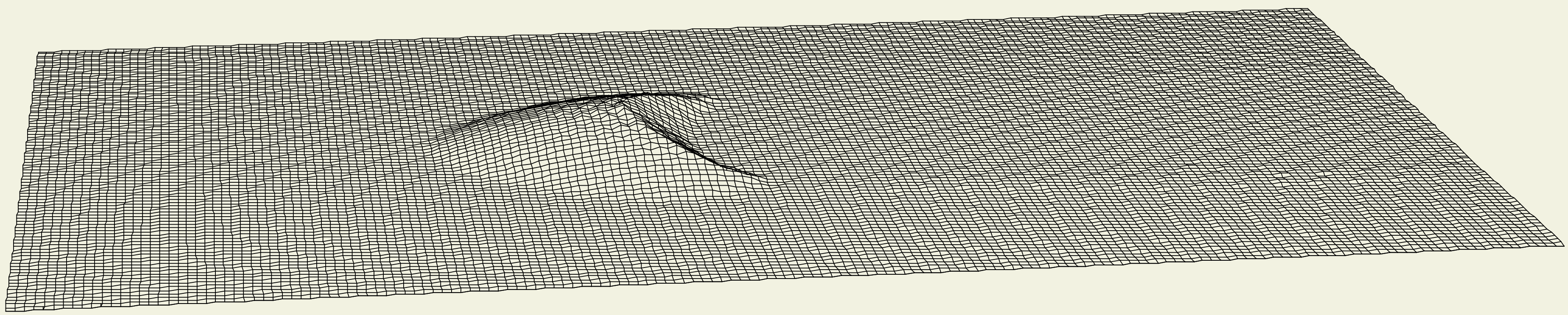

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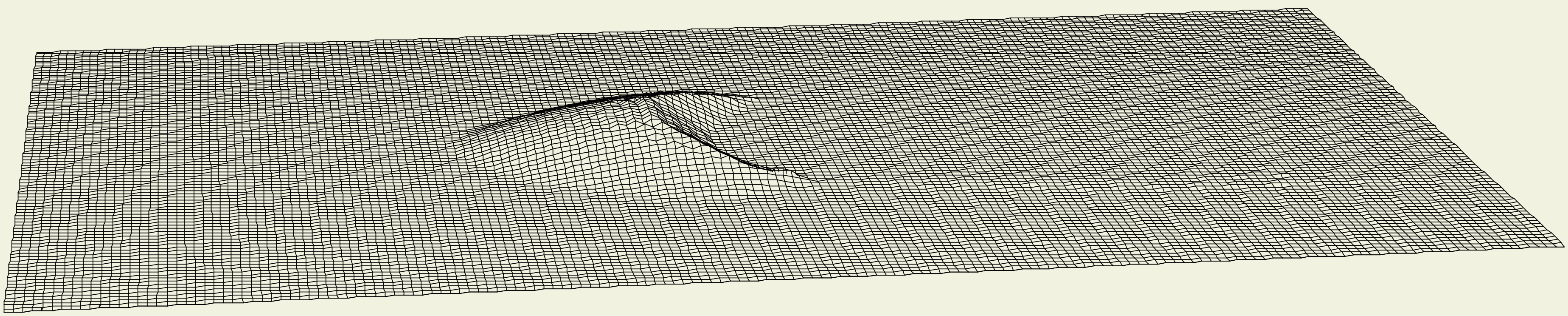

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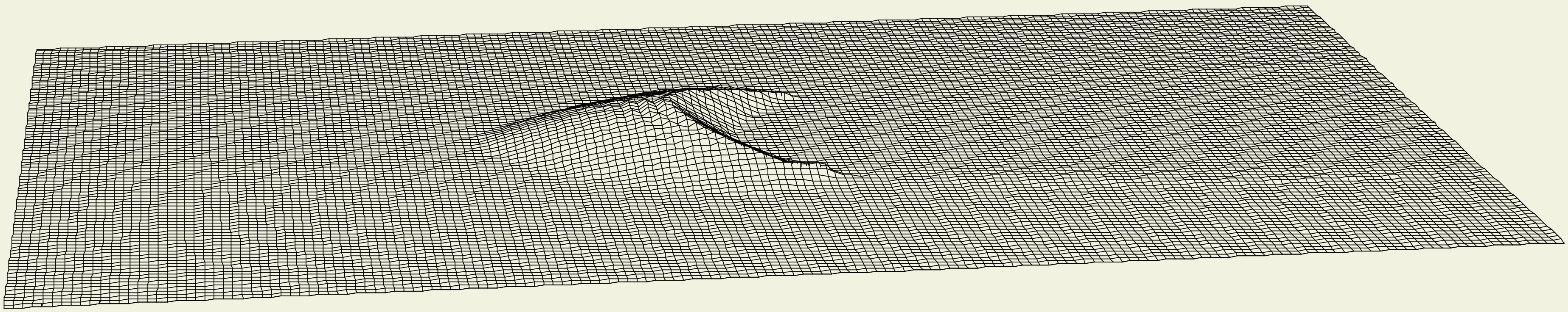

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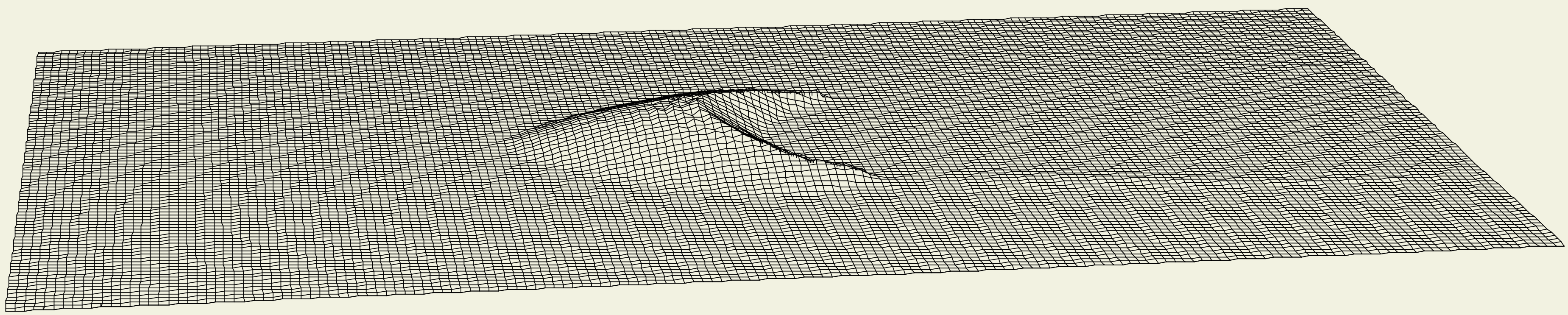

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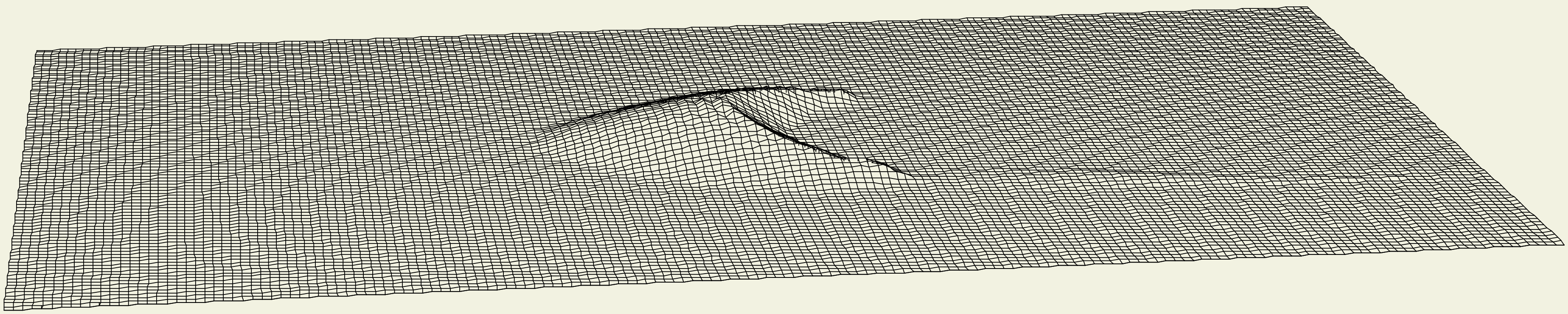
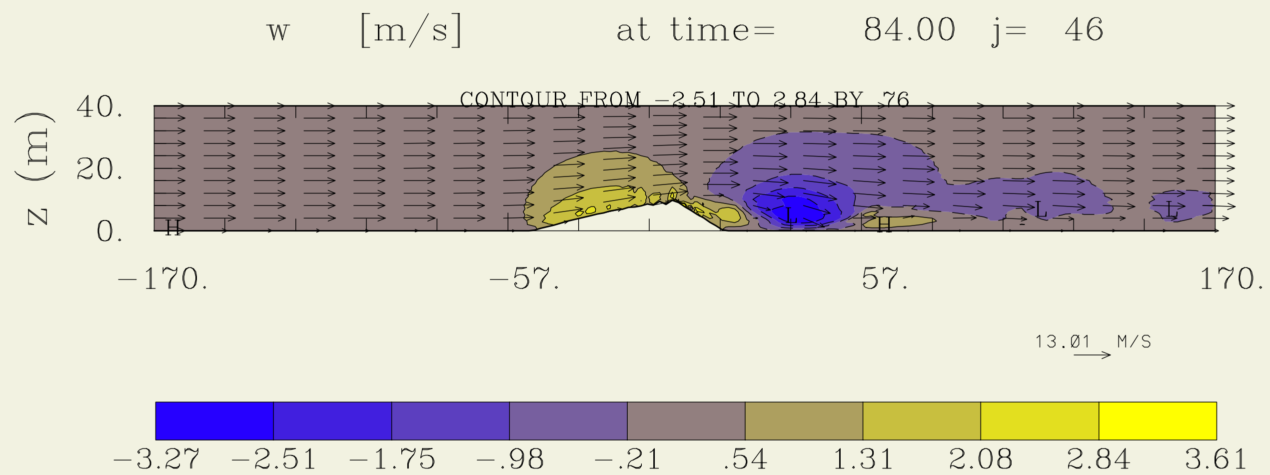
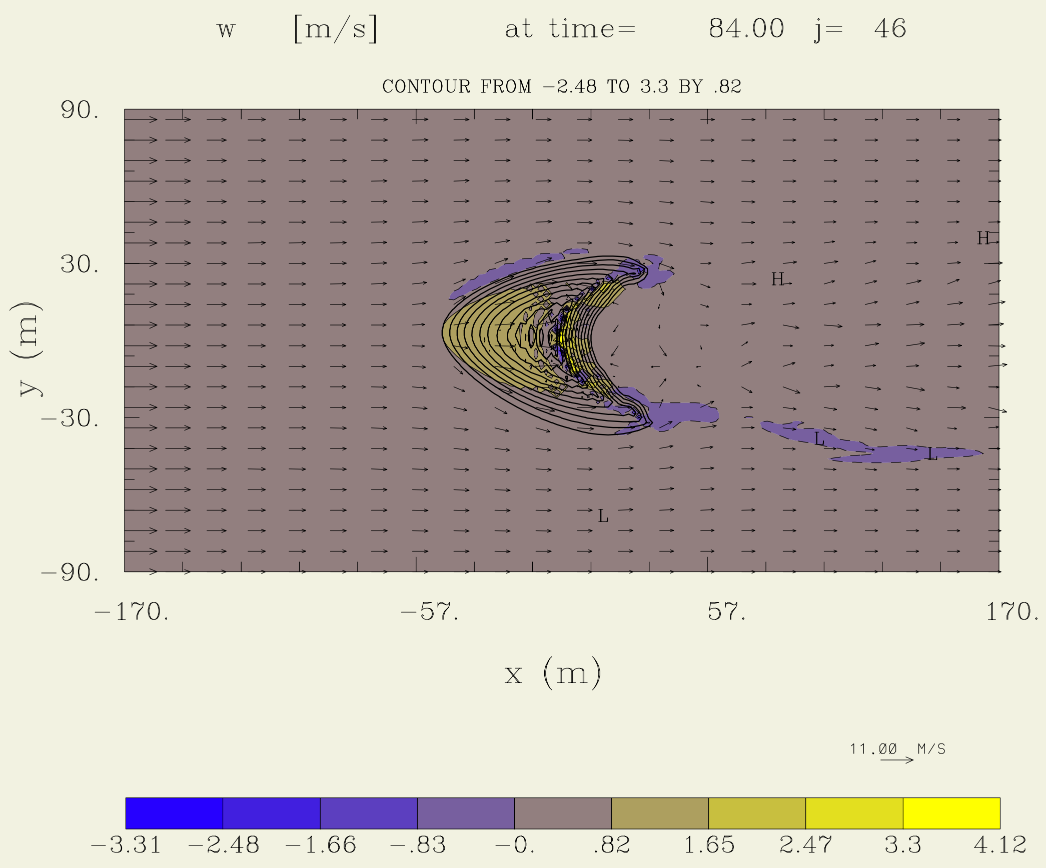


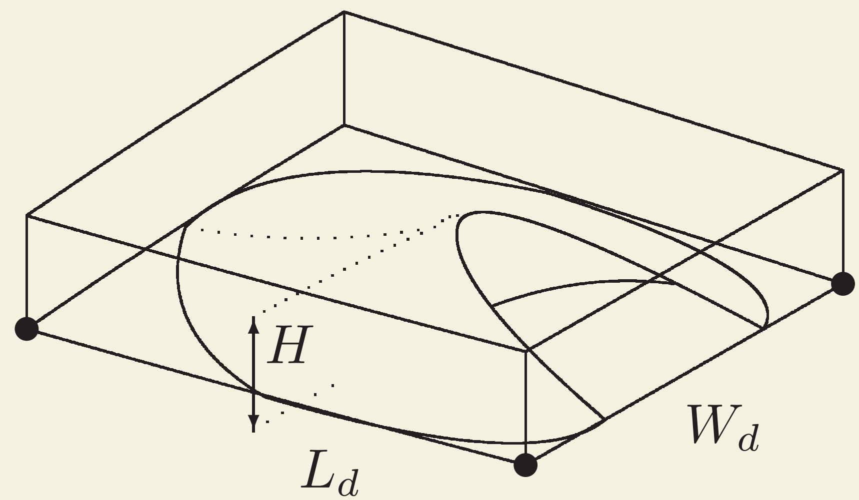
# Non erodible substrate xz



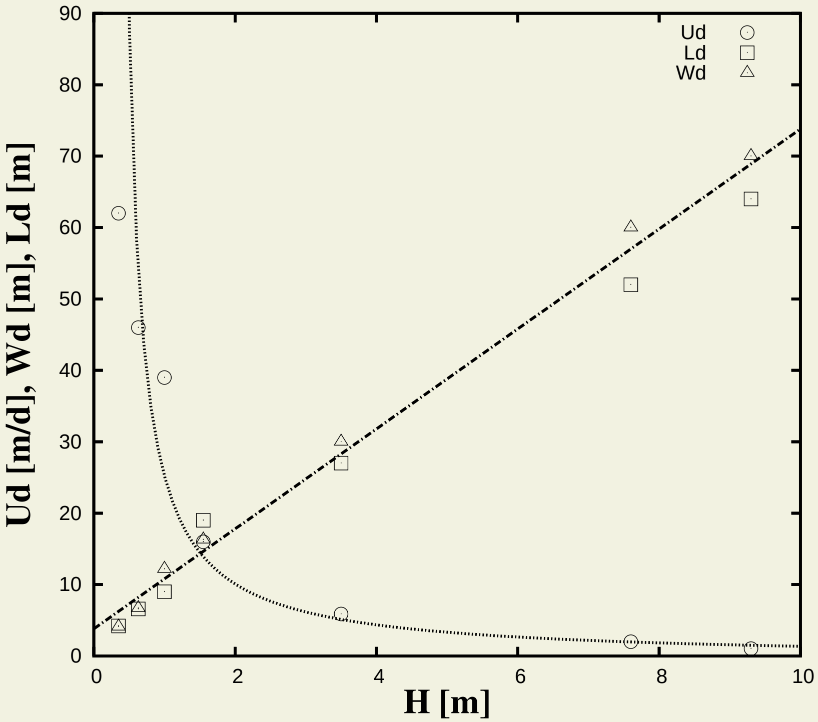
# Non erodible substrate xy



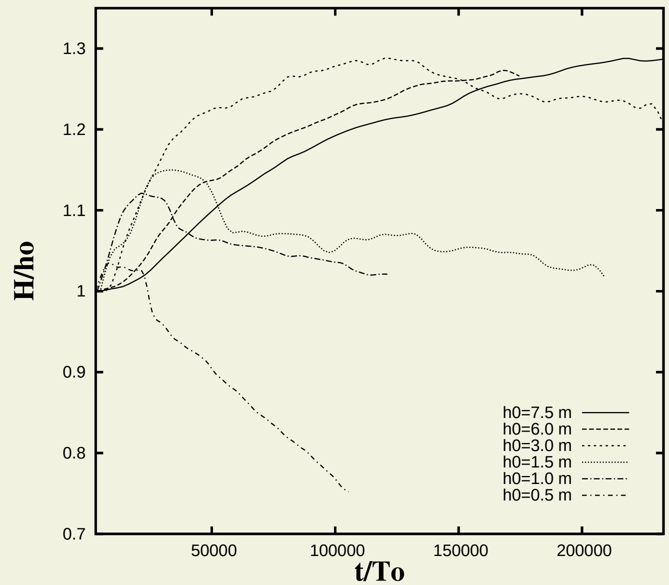
# Sketch of a dune. Dimensions $H$: height, $W_d$: maximal width, $L_d$: maximal length



# Non erodible substrate. Dune velocity, width and length



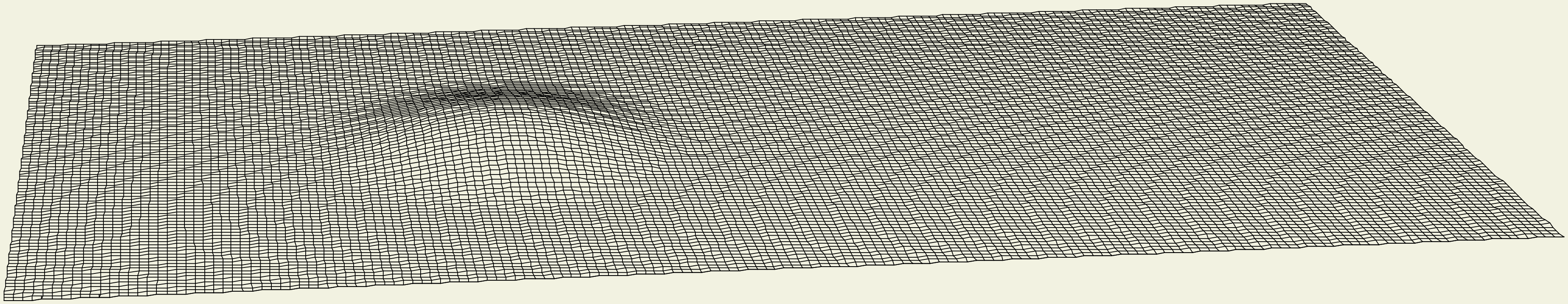
# Maximum height




$h_0$: Initial pile height, $T_0 = a/ \mid \mathbf{v}_e \mid$.

⋆ Stabilization of final heights for $H > 1$m and monotone decrease of heights for $H < 1$m (match observations!)

# Erodible basement. Profile



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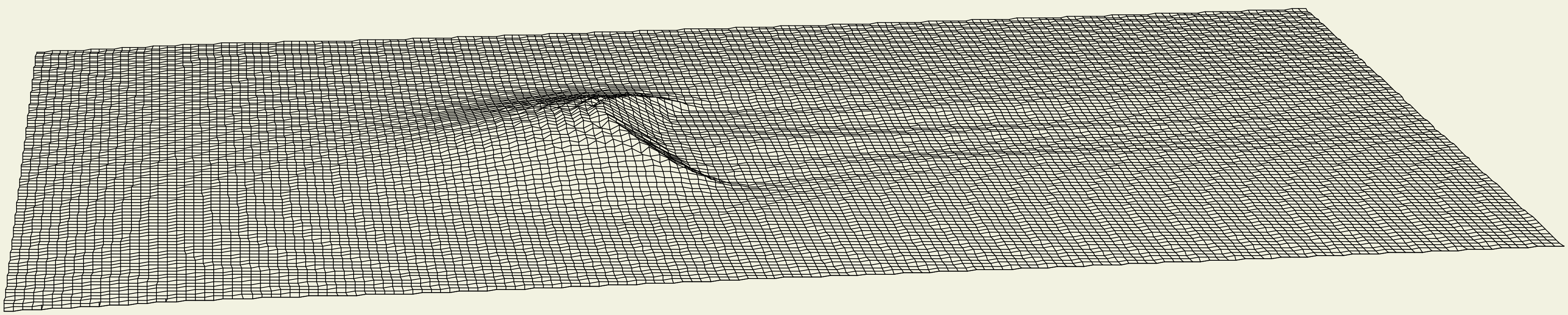

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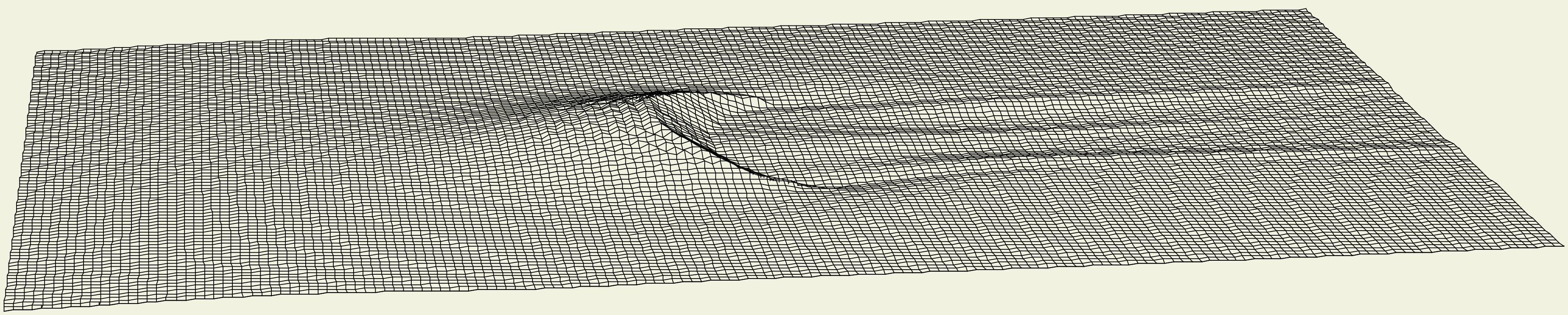

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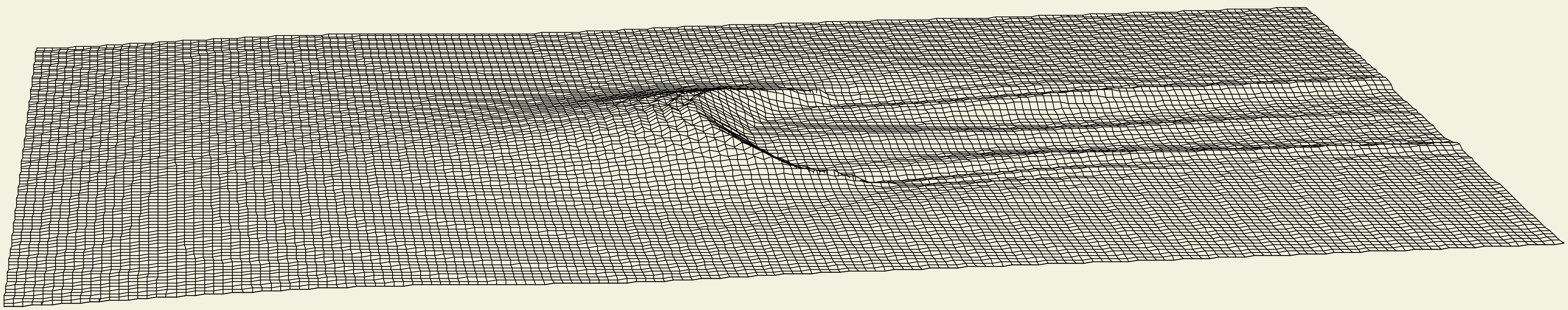
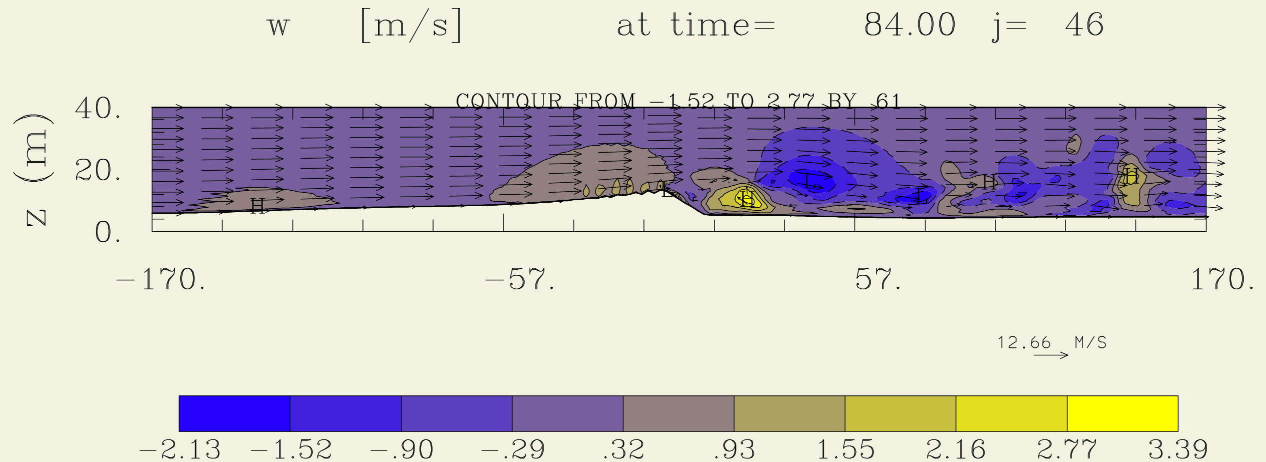
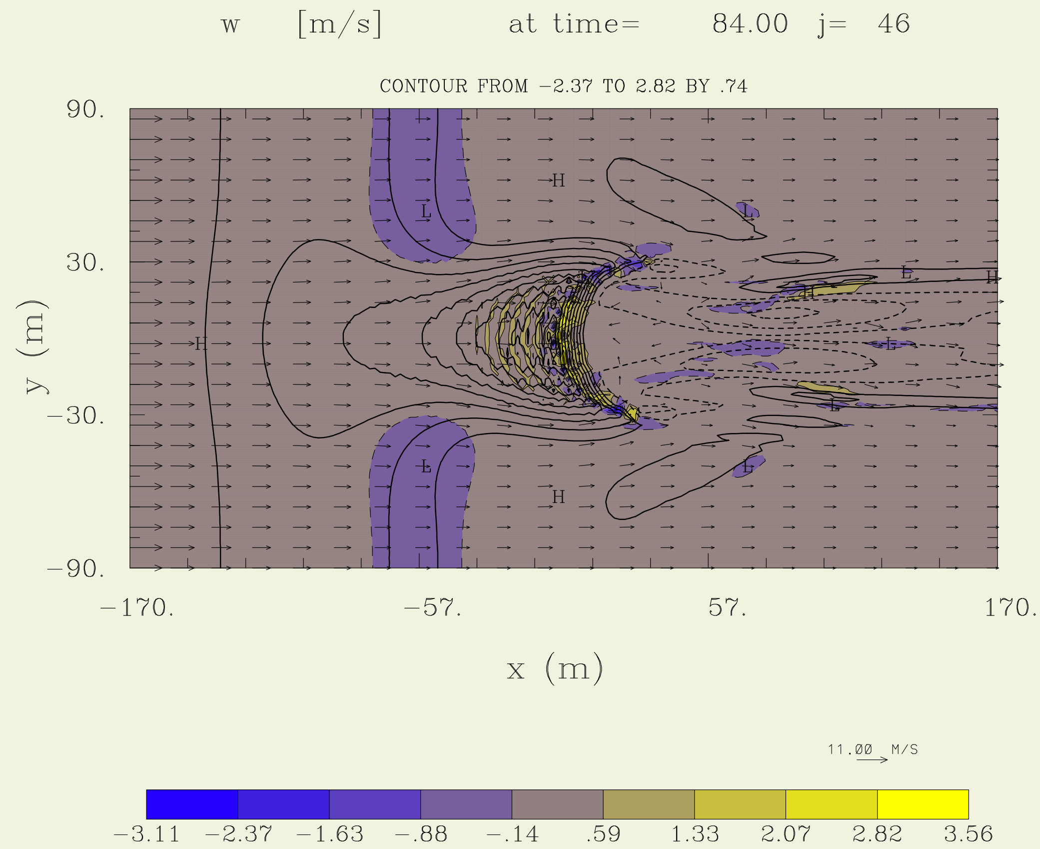


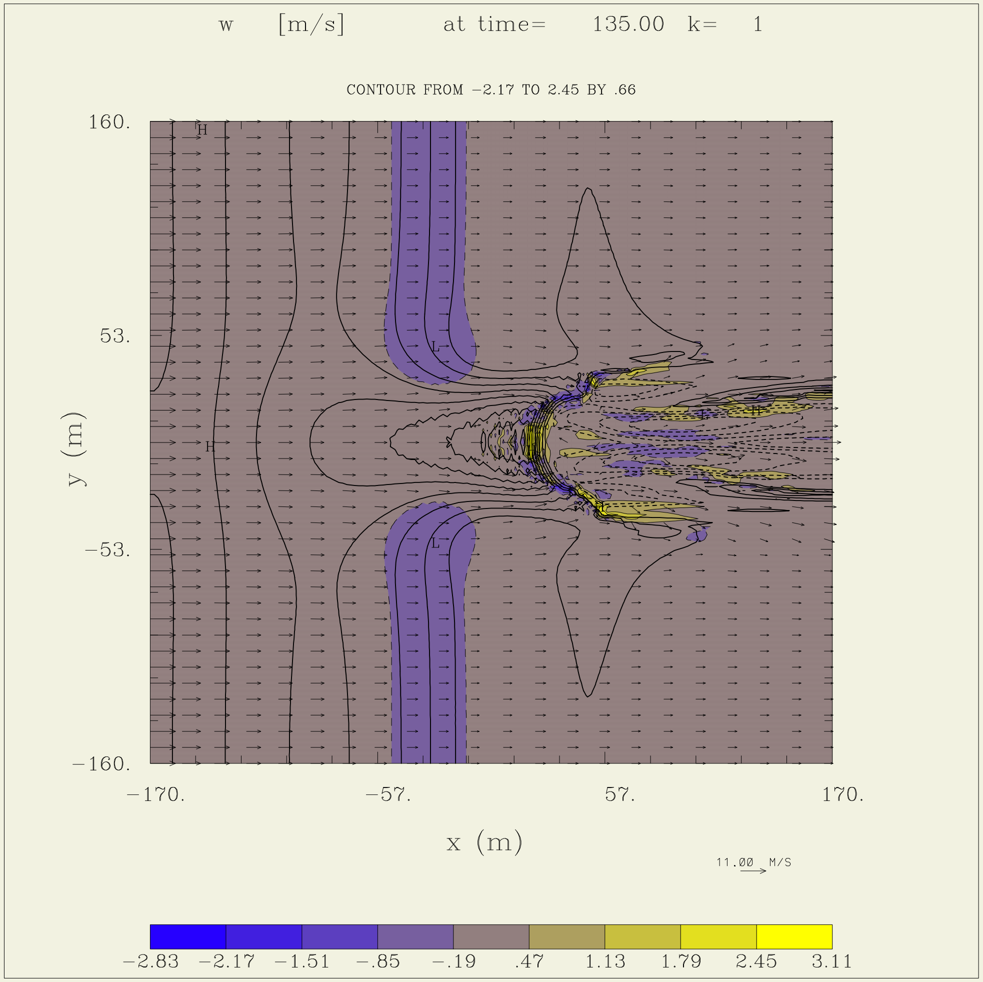
# Erodible substrate xz



# Erodible substrate xy



# Erodible substrate xy: Long term



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  Time dependent coordinate transformation, rescaling, LES, active landform.

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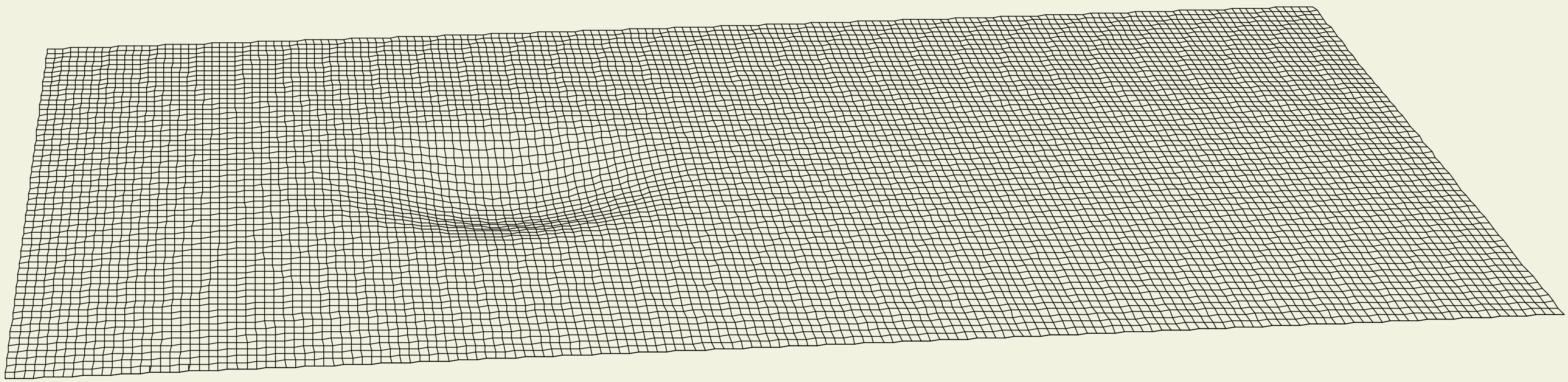

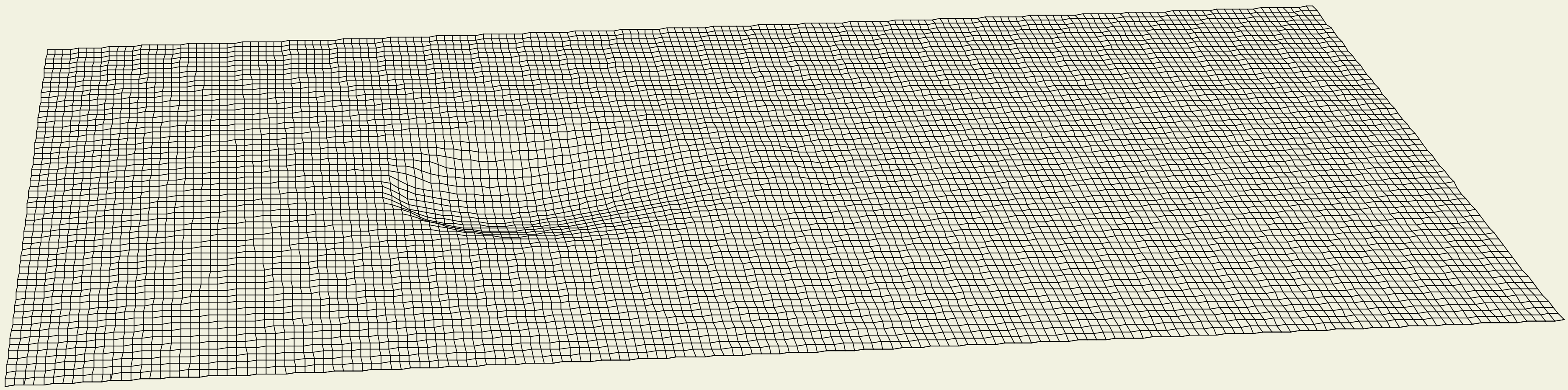

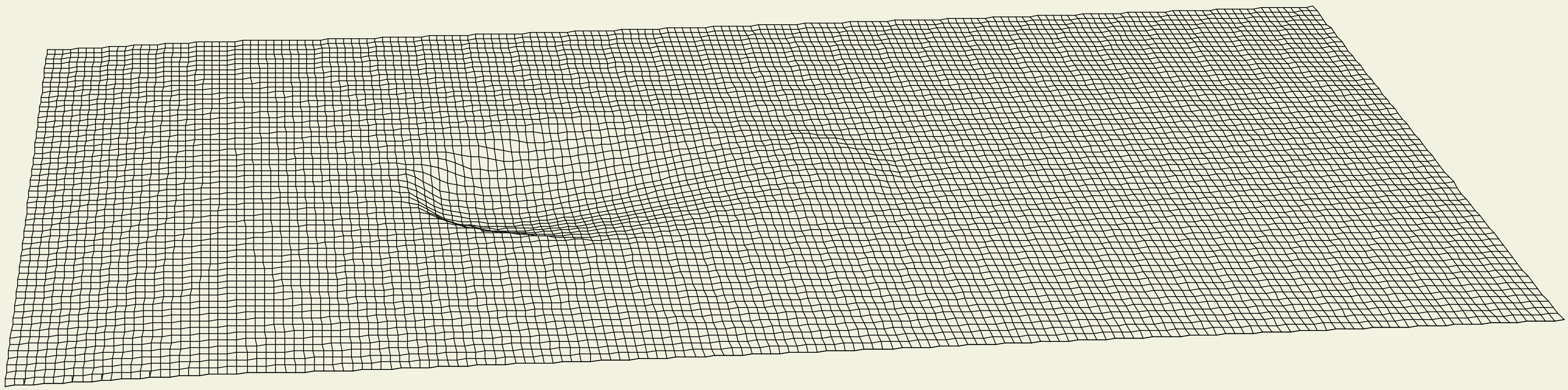

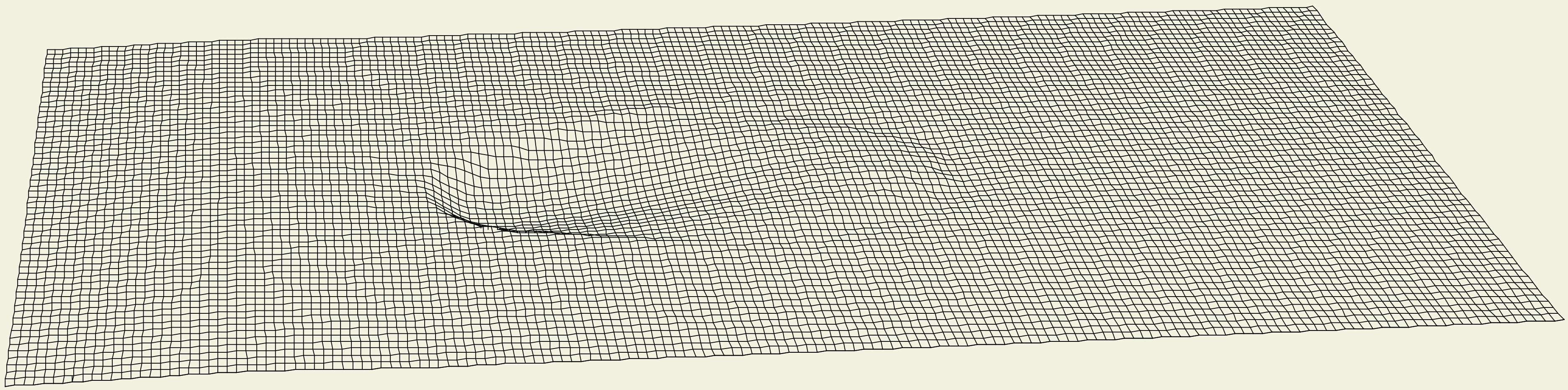

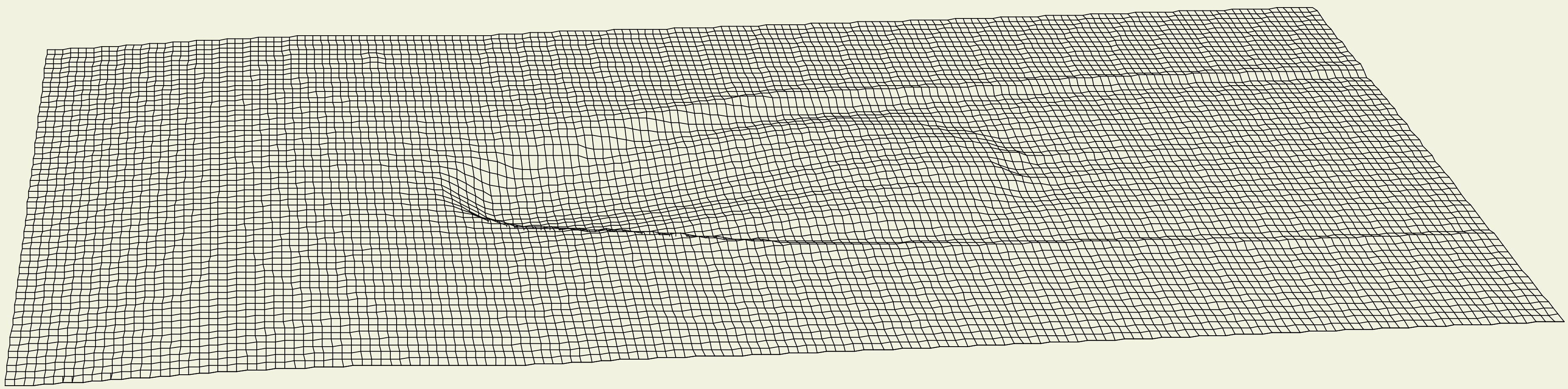

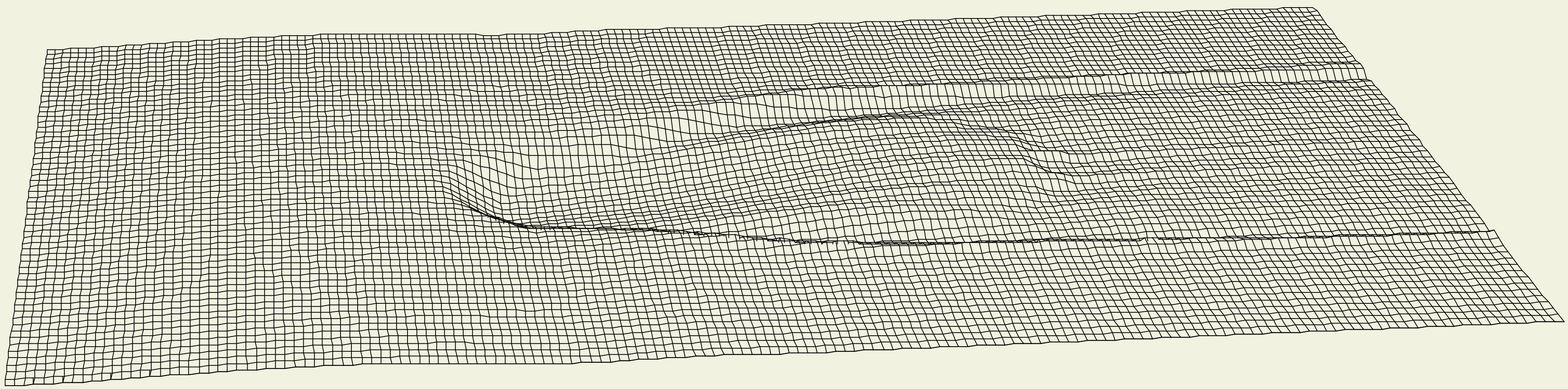

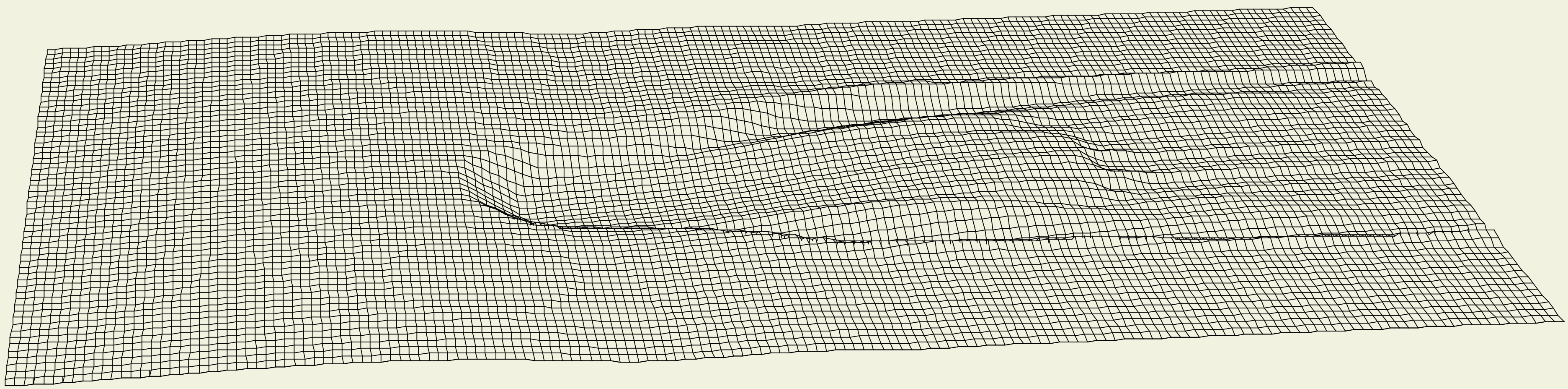

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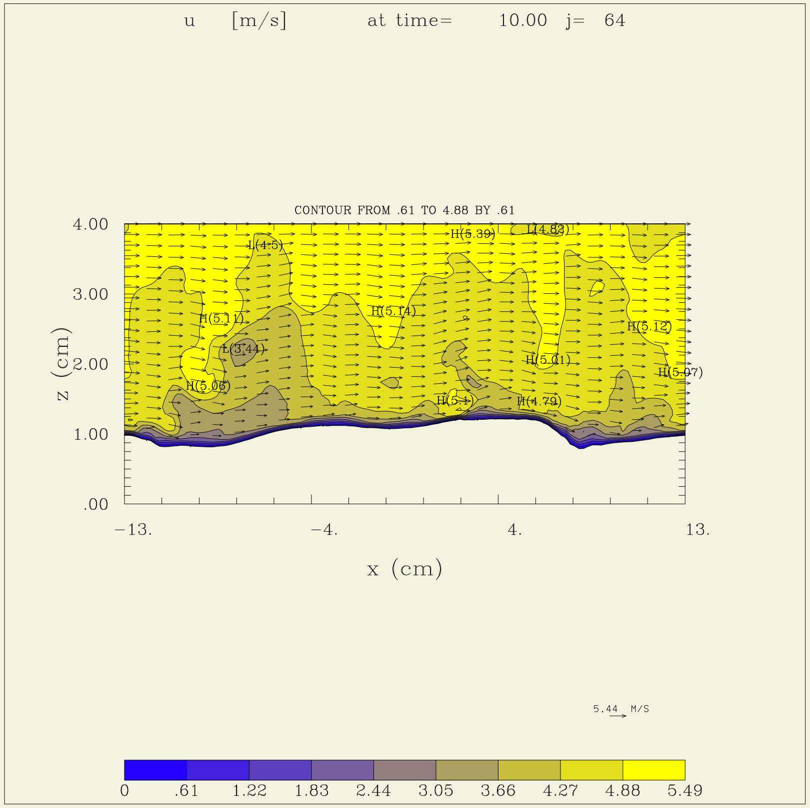
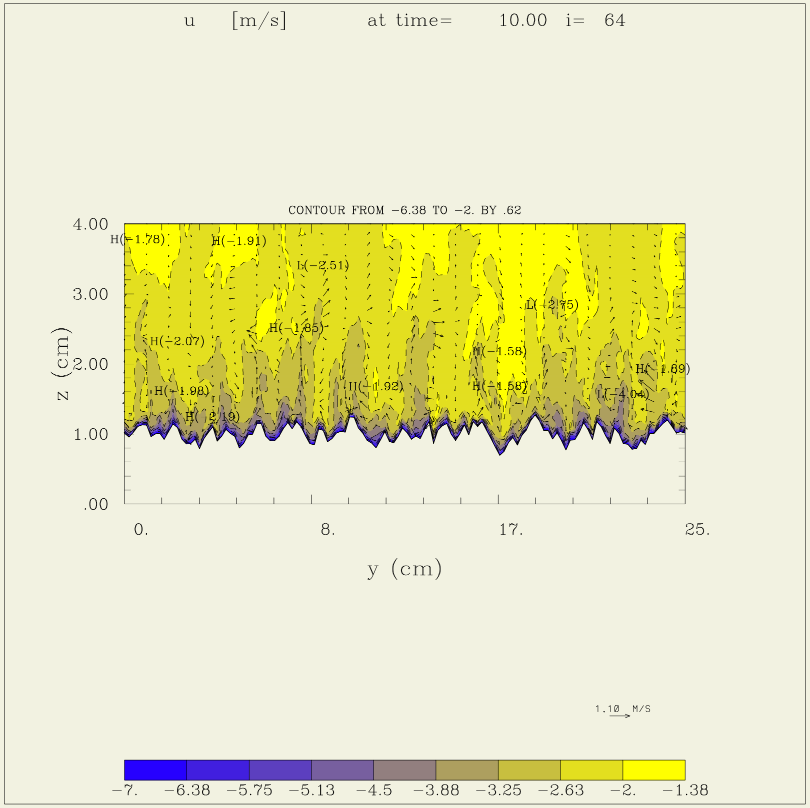
  - Small scales: ripples

# Ripple formation



# Ripple formation: stripped structures



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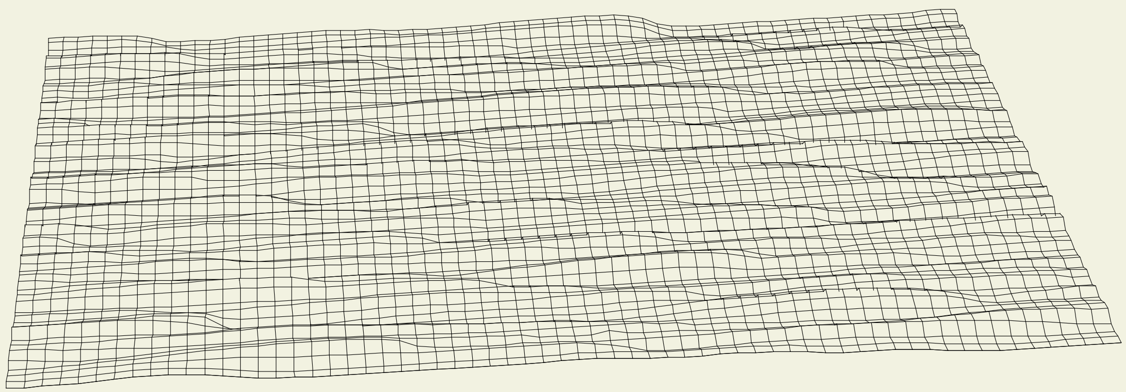

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  - Local scales: evolution of simple forms. Dependence on initial conditions
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- b) Dynamics of more complex flows:
  - Rotating
  - Stratified and thermally forced
  - Free surface flows