Gravity-wave breaking beyond traditional instability concepts

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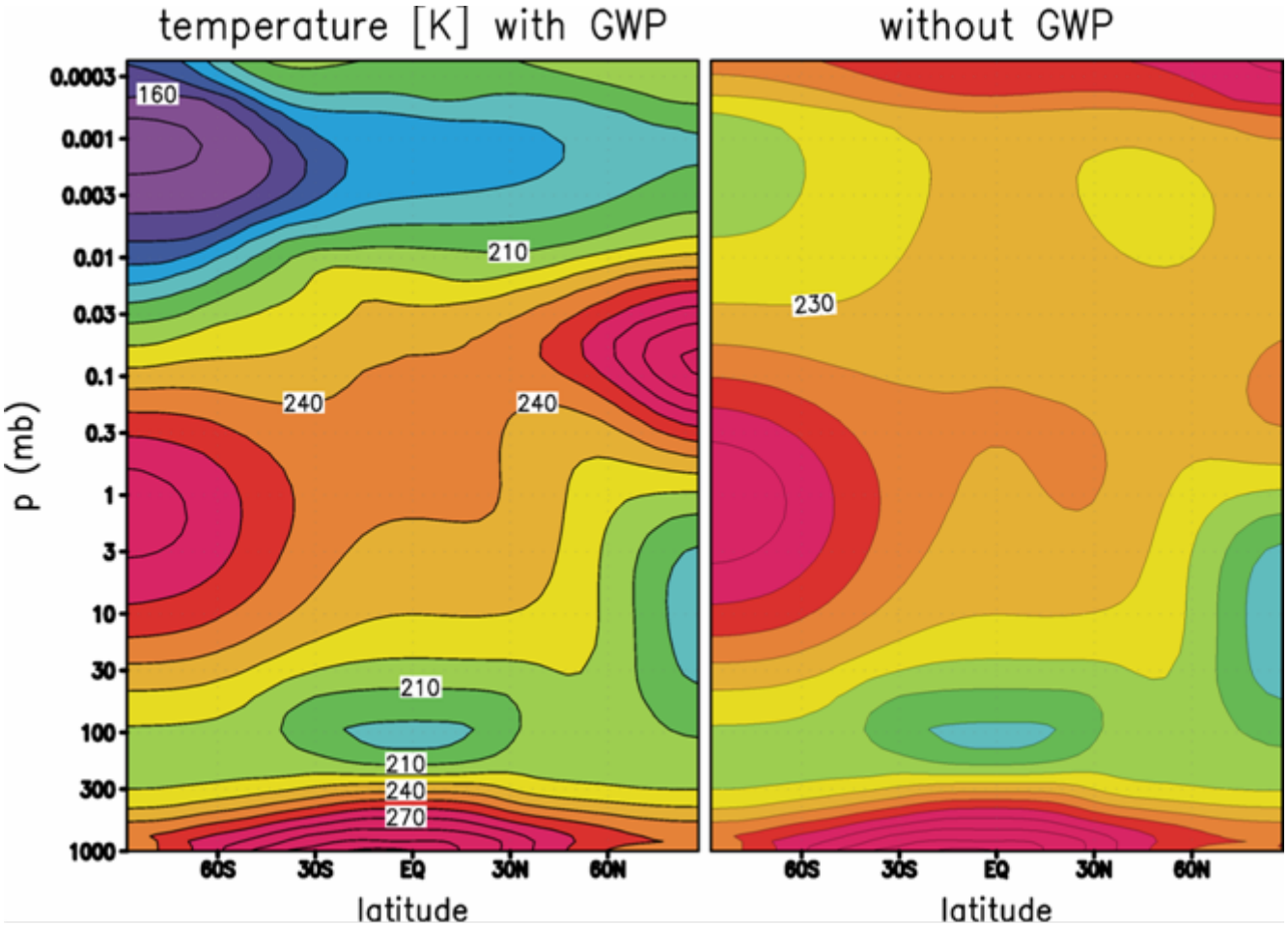
Gravity Waves in the Middle Atmosphere



Becker und Schmitz (2003)

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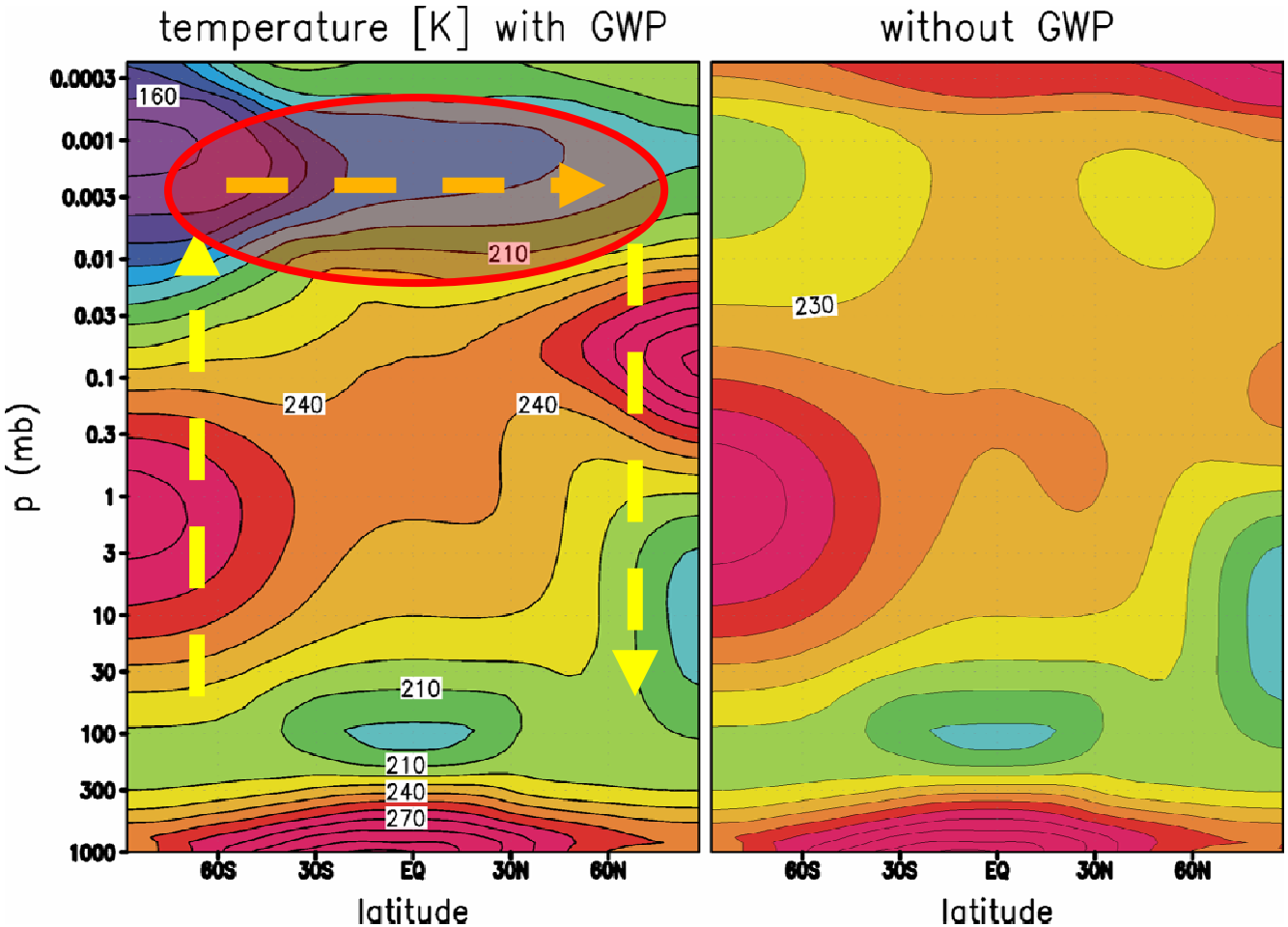

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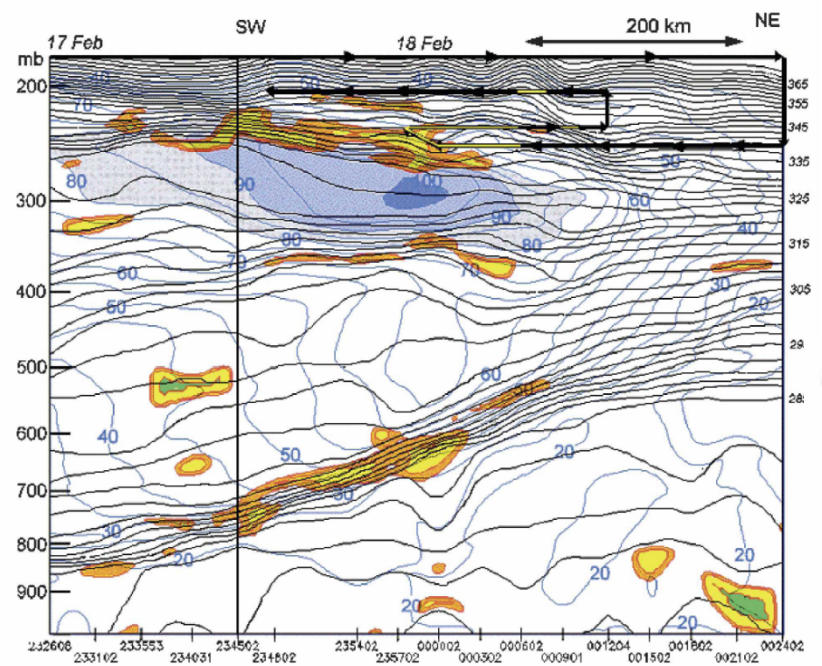
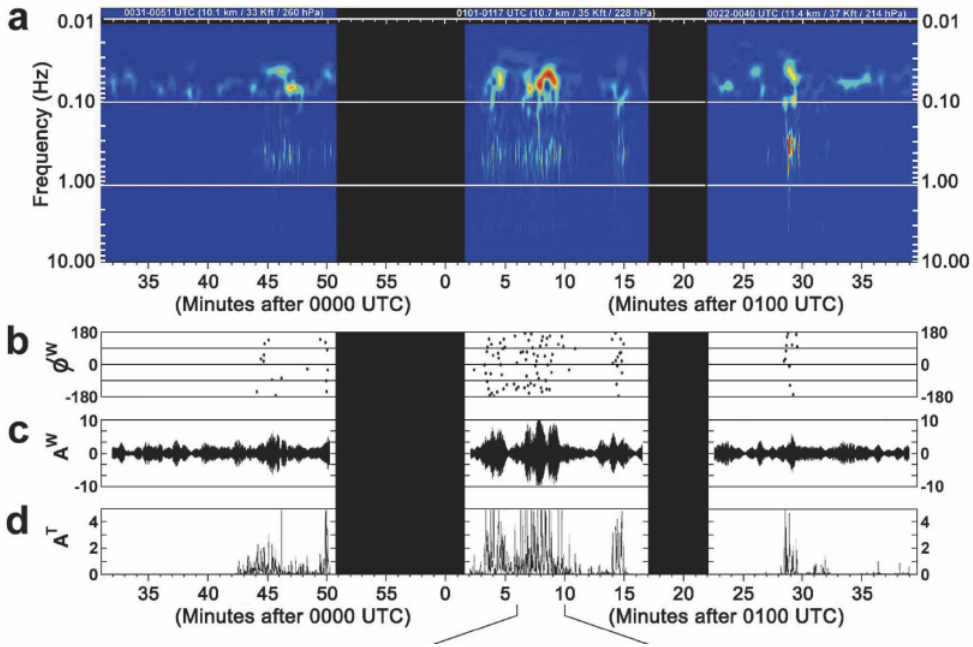
Gravity Waves and Clear-Air Turbulence



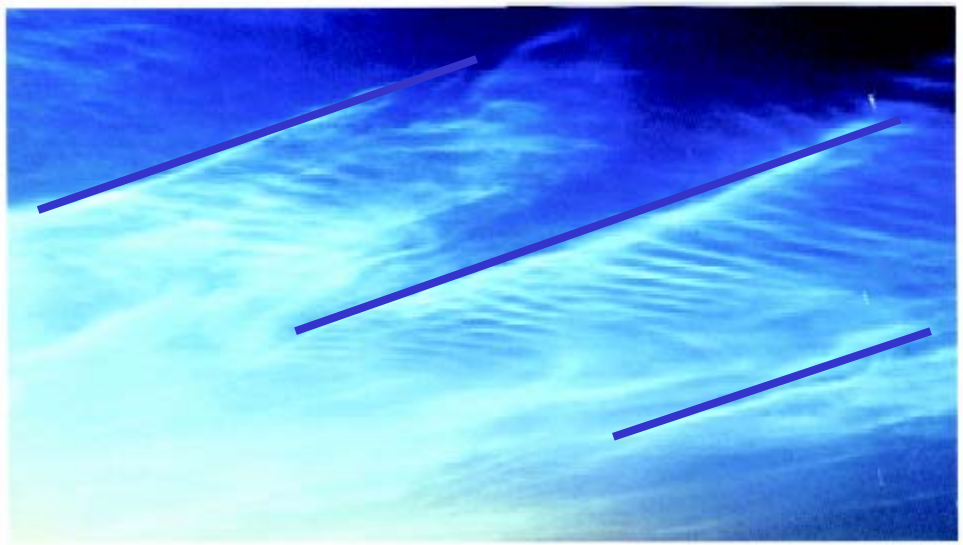


Koch et al (2008)

GW Parameterizations

- multitude of parameterization approaches (Lindzen 1981, Medvedev und Klaassen 1995, Hines 1997, Alexander and Dunkerton 1999, Warner and McIntyre 2001)
- too many free parameters
- reasons :
  - …
  - insufficent knowledge: ***conditions of wave breaking***
- basic paradigm: ***breaking of a single wave***
  - Stability analyses (NMs, SVs)
  - Direct numerical simulations (DNS)

Basic Types of GWs



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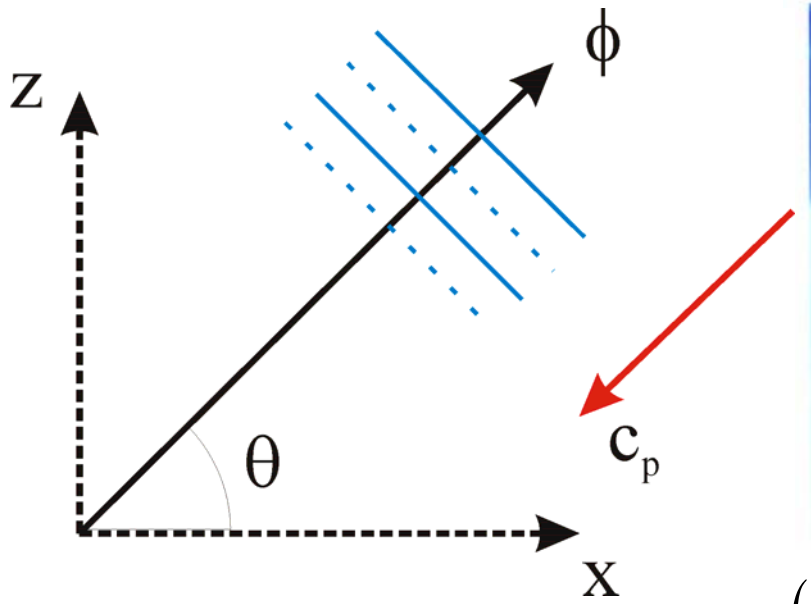

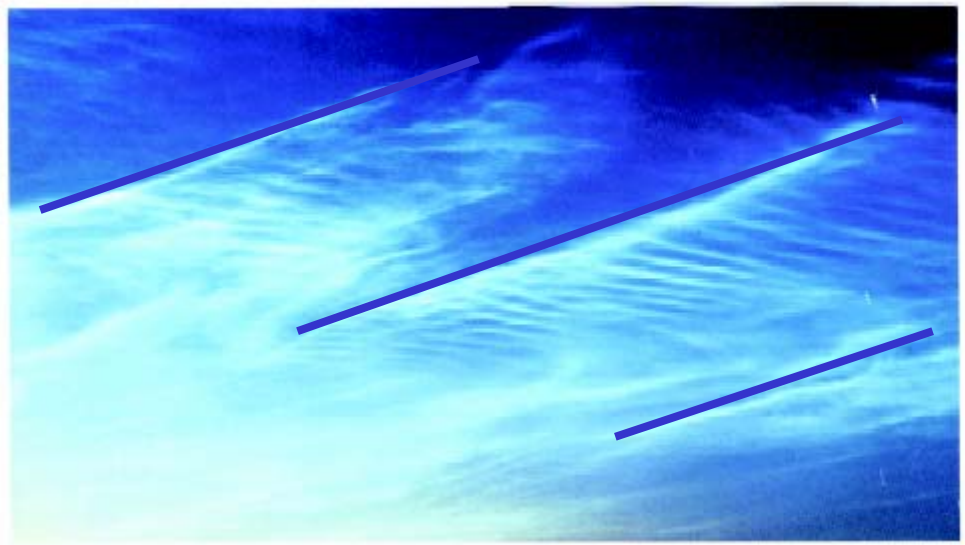

- Dispersion relation: $\omega = \pm\left(f^2 \sin^2\Theta + N^2 \cos^2\Theta\right)^{1/2}$

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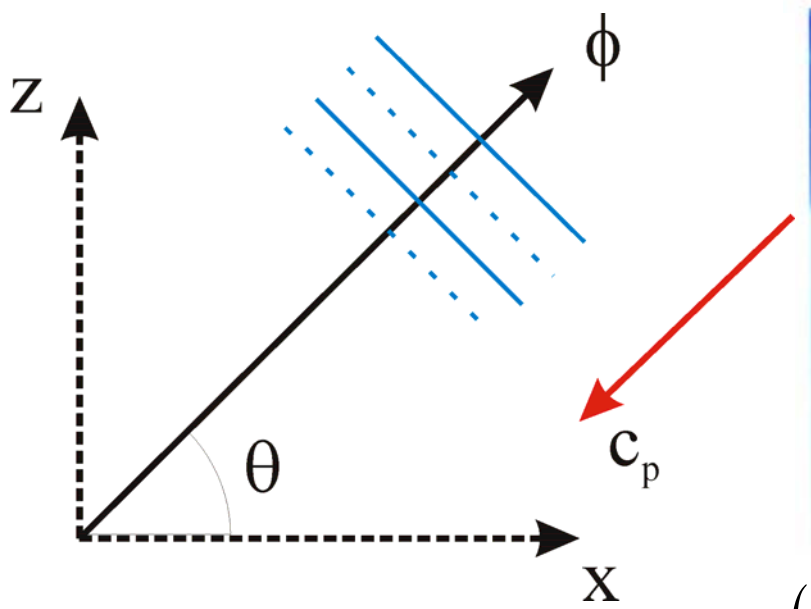

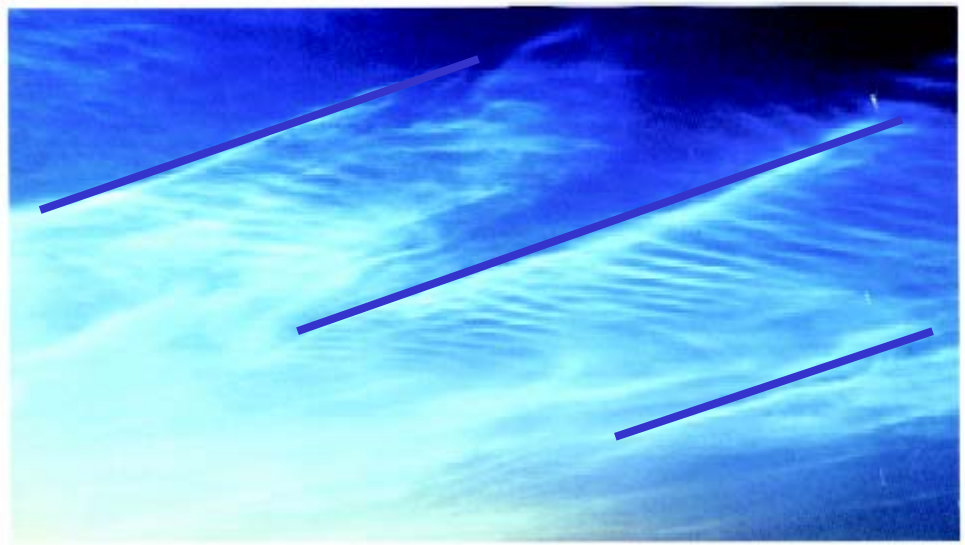

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- ***Inertia-gravity waves (IGW):***

$$\Theta \approx 90^\circ \quad \Rightarrow \quad \omega \approx \pm f\left(1 + \frac{N^2}{2f^2}\cot^2\Theta\right)$$

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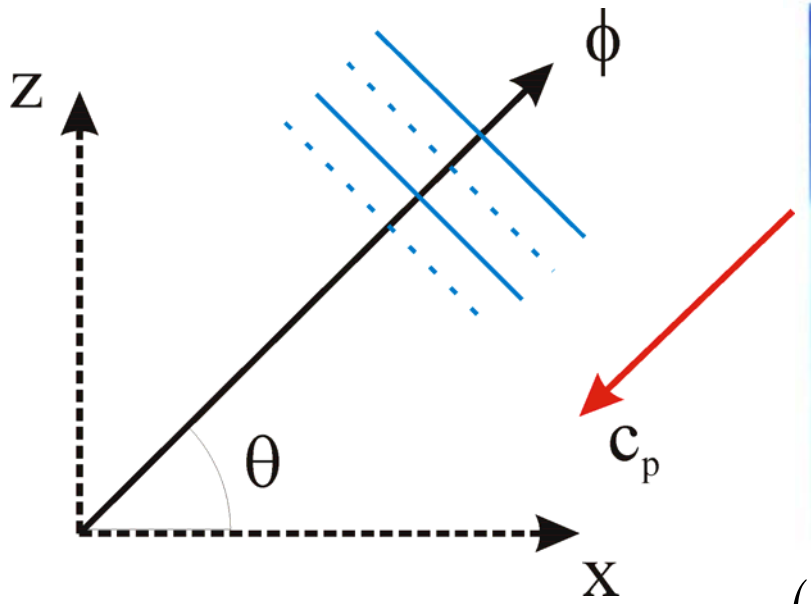

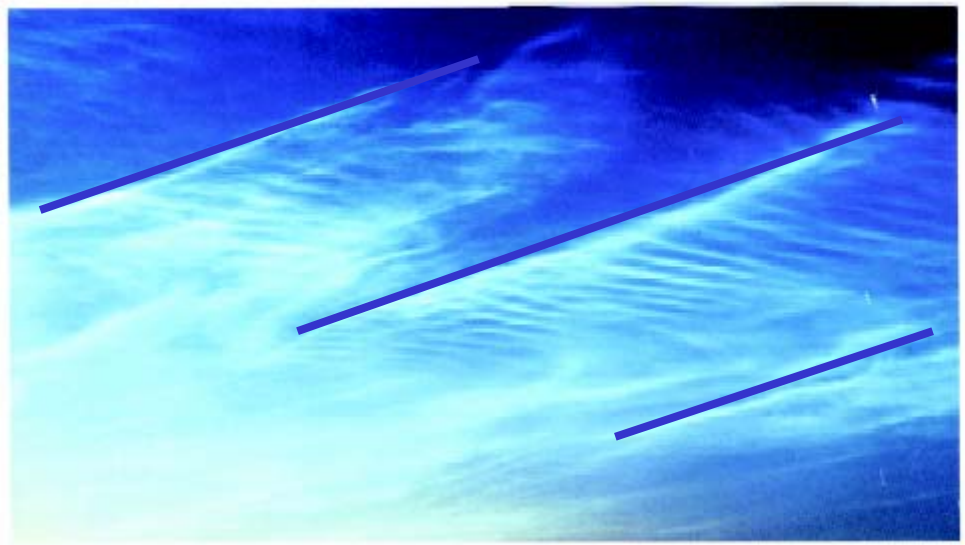

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- ***High-frequency gravity waves (HGW):***

$$\Theta < 90^\circ \quad \Rightarrow \quad \omega \approx \pm N\cos\Theta$$

Traditional Instability Concepts

- ***static (convective) instability:***

$$\frac{\partial B_{tot}}{\partial z} = N^2 + \frac{\partial b}{\partial z} < 0$$

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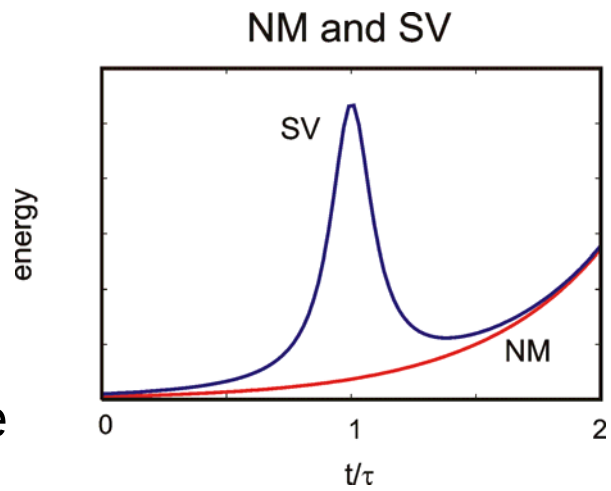
- ***BUT: limited applicability to GW breaking***

GW Instabilities Beyond Traditional Concepts

IGW:

- ***NM:*** Exponential energy growth simple shear layer:
  - Static instability if $N_{tot}^2 = N^2 + \partial b / \partial z < 0$
  - Dynamic instability if $\mathrm{Ri} = N_{tot}^2 / \left[(\partial u / \partial z)^2 + (\partial v / \partial z)^2 \right] < 1/4$

- ***SV:*** optimal energy growth over a finite time (Farrell 1988a,b, Trefethen et al. 1993)



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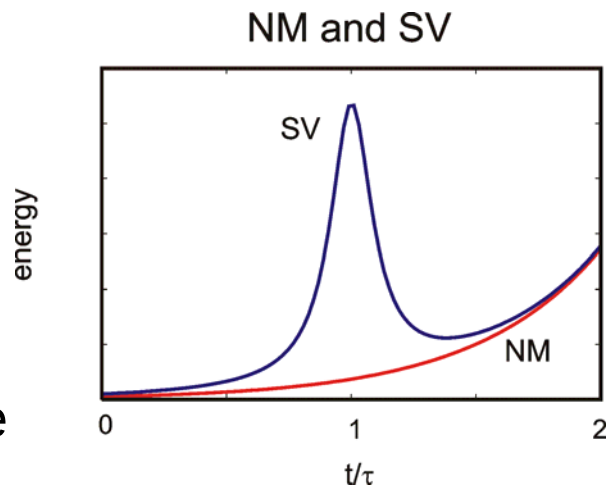

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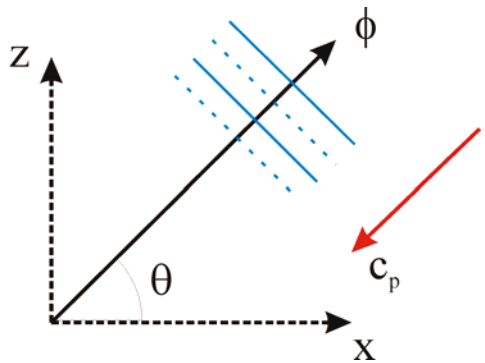
HGW:

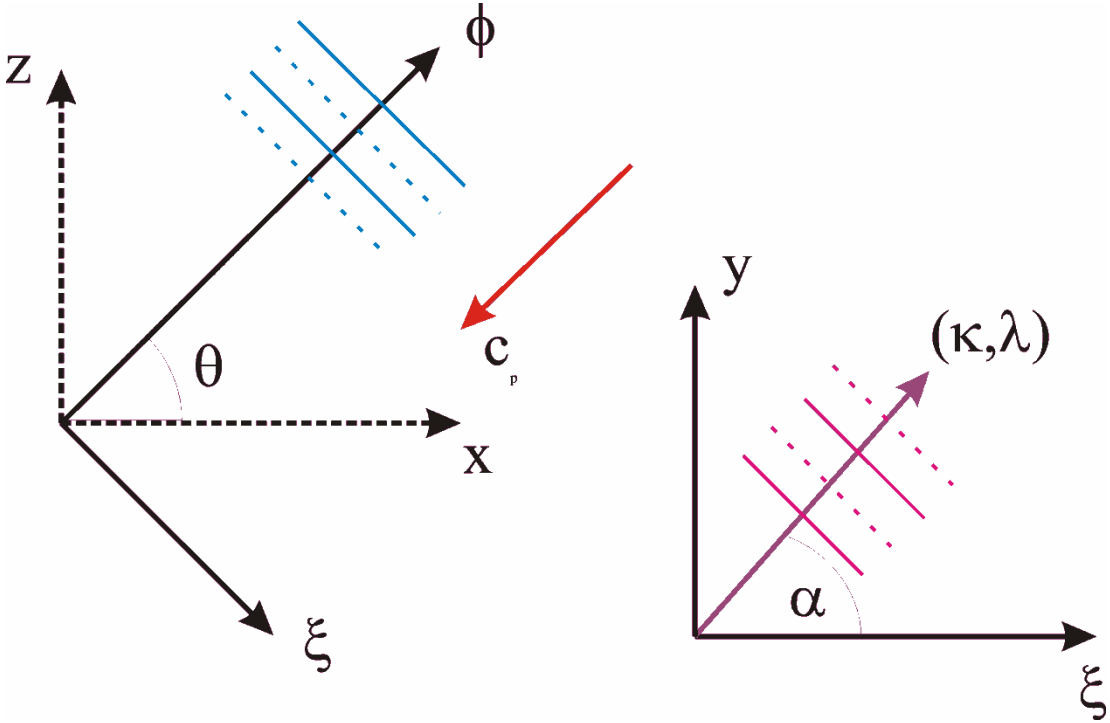
- Significant ***horizontal gradients***

  $\Rightarrow$

  shear-layer concepts fail
  (e.g. Lombard and Riley 1996, Achatz 2005)

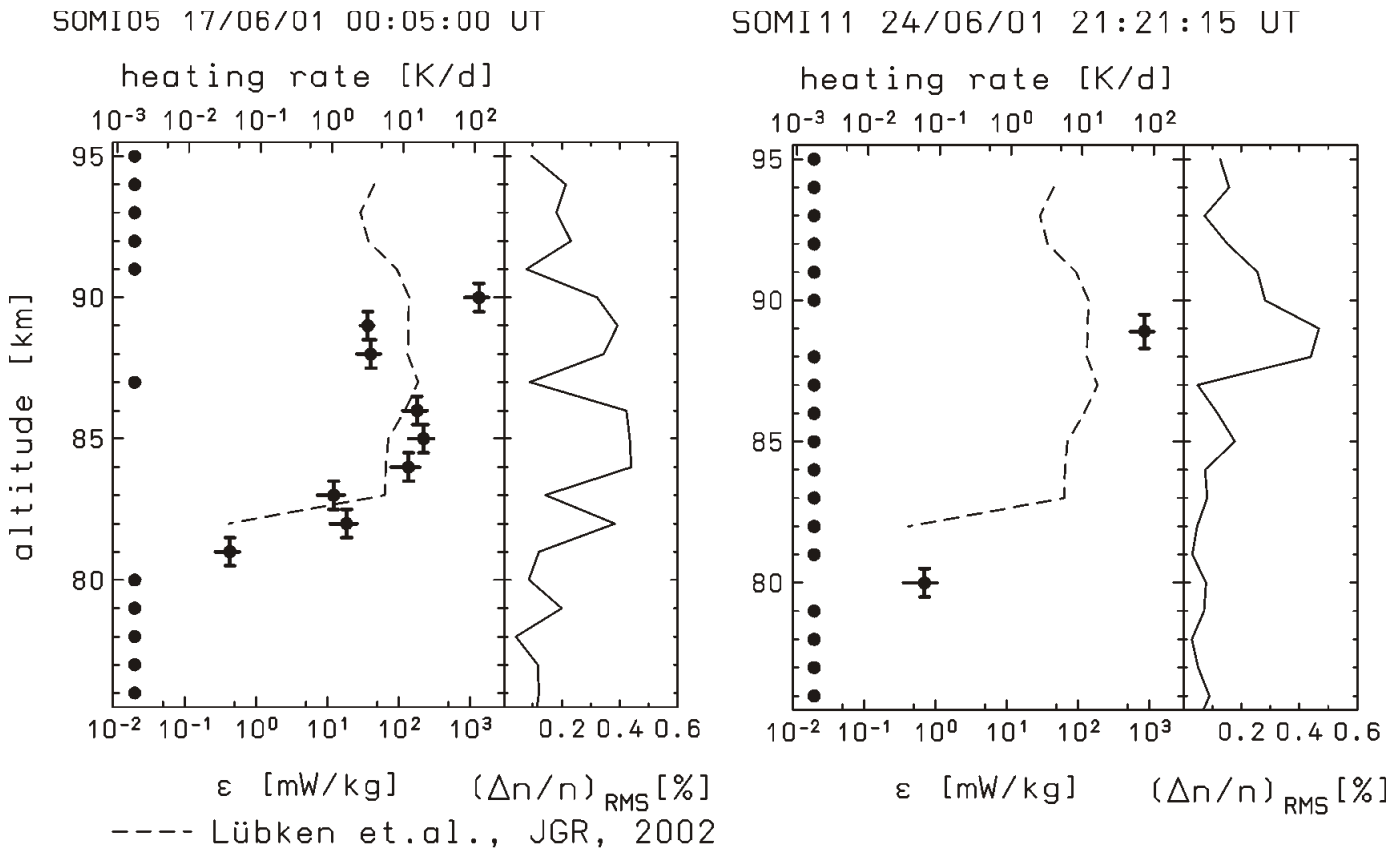


Stability analysis, 2.5D-DNS



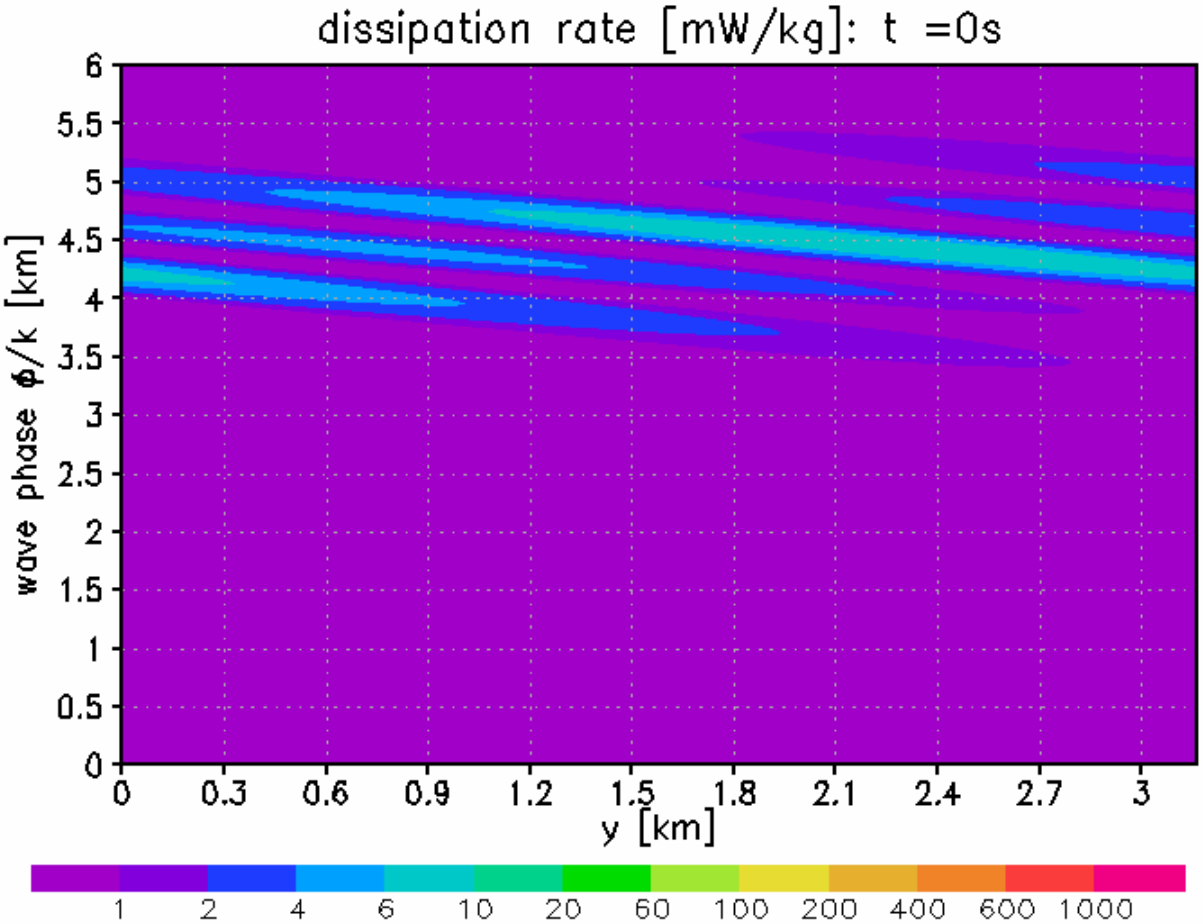
$$\begin{pmatrix} \mathbf{v} \\ b \end{pmatrix}(\xi, y, \phi, t=0) = \begin{pmatrix} \mathbf{v} \\ b \end{pmatrix}_{\mathrm{SW}}(\phi) + \begin{pmatrix} \mathbf{v} \\ b \end{pmatrix}_{\mathrm{NM,SV}}(\phi) \exp\left[i \underbrace{(\kappa\xi + \lambda y)}_{kx_{\parallel}} \right]$$

Turbulent Dissipation Rates in the Mesosphere



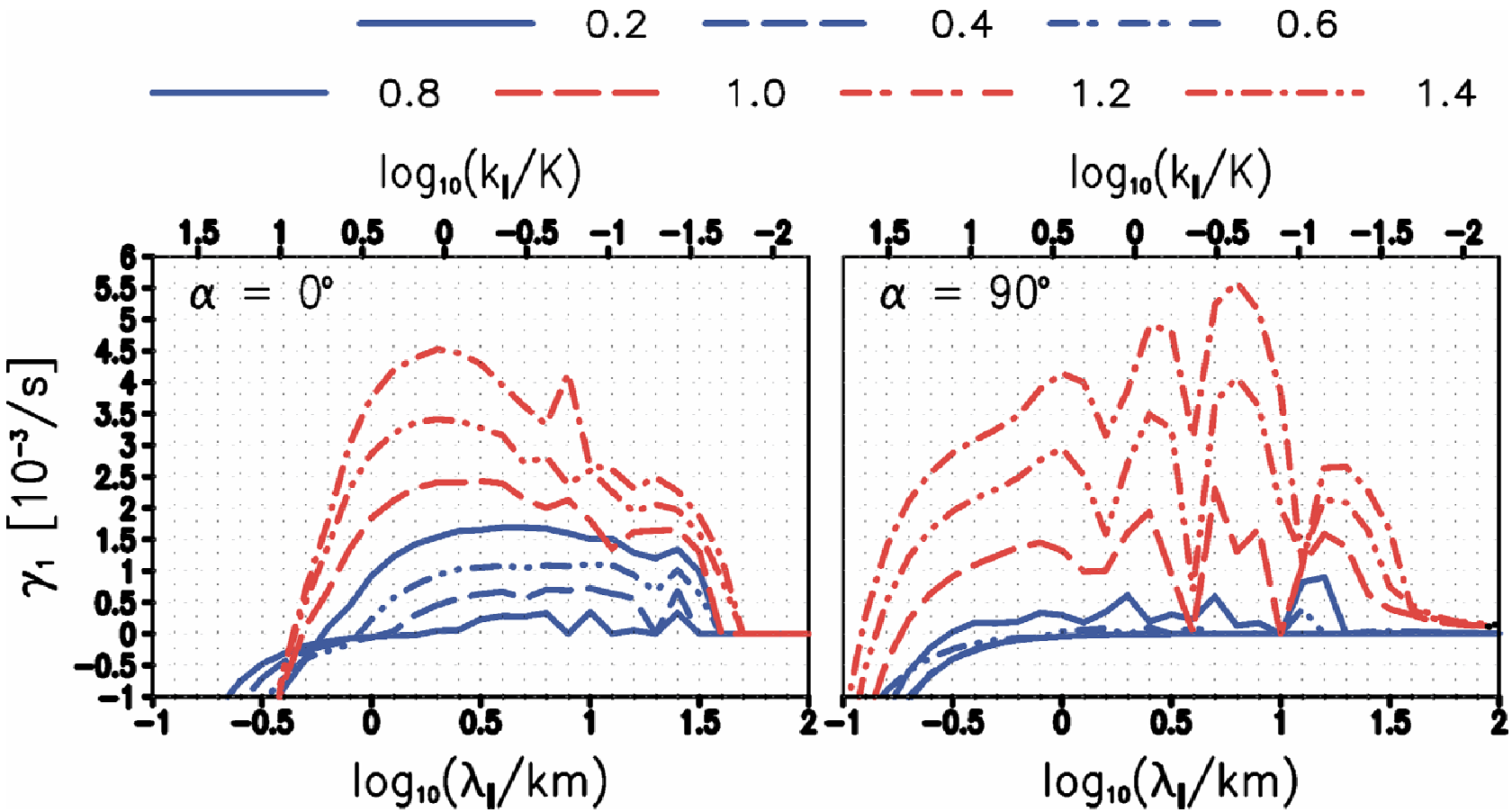
Müllemann et al., 2003

Breaking of an IGW by an SV (Ri > ¼):

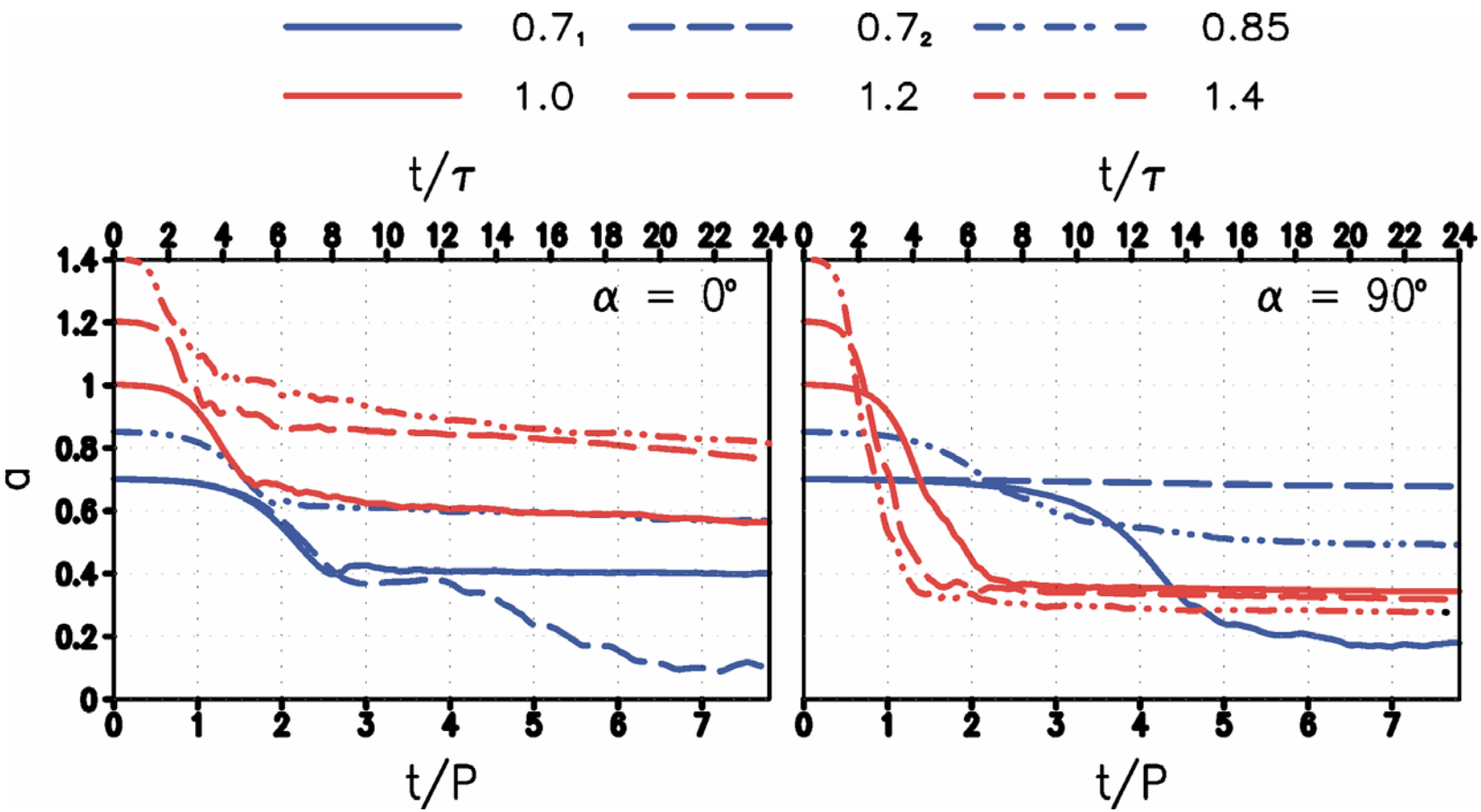


Measured dissipation rates 1…1000 mW/kg (Lübken 1997, Müllemann et al. 2003)

NMs of an HGW: Growth Rates ($\Theta = 70°$)



HGW amplitude after a perturbation by a NM (DNS):



Energetics

- average over ξ and $y \rightarrow \left(\overline{\mathbf{v}}, \overline{b}\right)$
- deviation $\rightarrow \left(\mathbf{v}', b'\right)$

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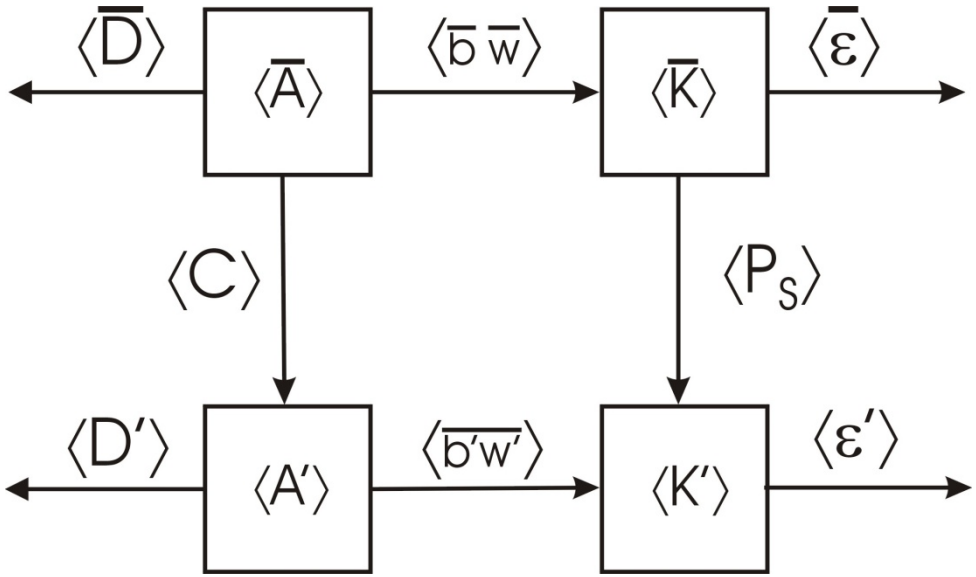

$$K = \frac{1}{2}|\mathbf{v}|^2 \qquad A = \frac{b^2}{2N^2}$$

$$P_S = -\overline{\mathbf{v}' u'_\phi} \cdot k \frac{d\bar{\mathbf{v}}}{d\phi} \qquad C = -\overline{b' u'_\phi} k \frac{d\bar{b}}{d\phi}$$

not the gradients in z matter,
but those in ϕ!

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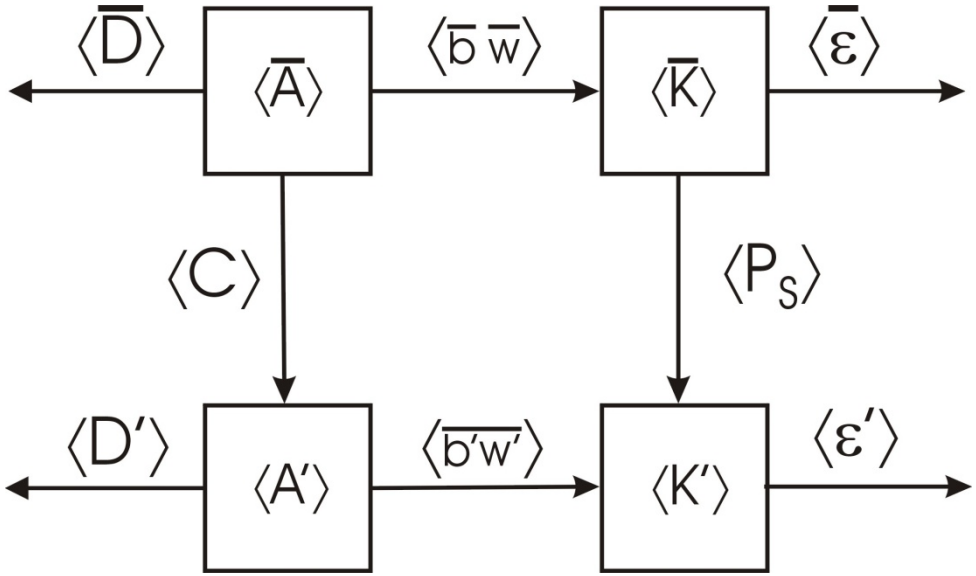

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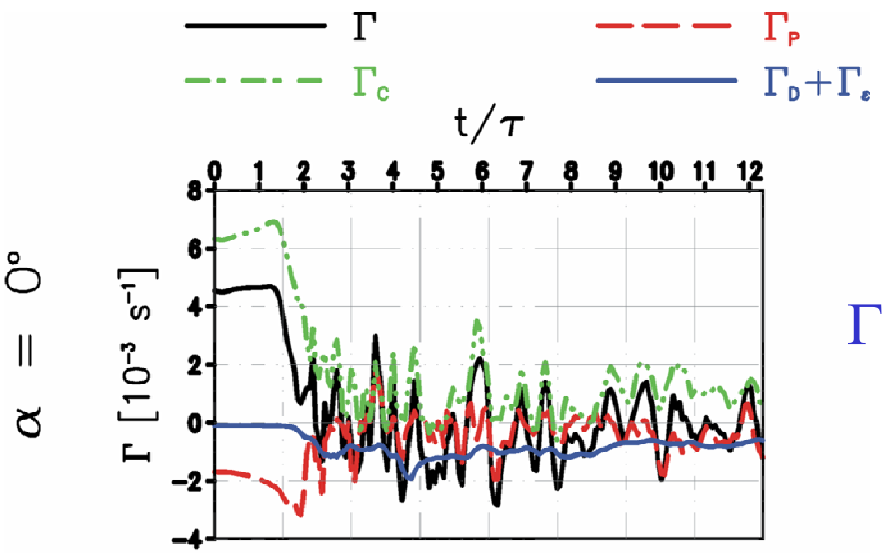
instantaneous growth rate:

$$E' = K' + A' \Rightarrow \Gamma = \frac{d\left\langle E'\right\rangle / dt}{2\left\langle E'\right\rangle} = \Gamma_P + \Gamma_C + \Gamma_D + \Gamma_\varepsilon$$

Energetics of a breaking HGW with $a_0 > 1$



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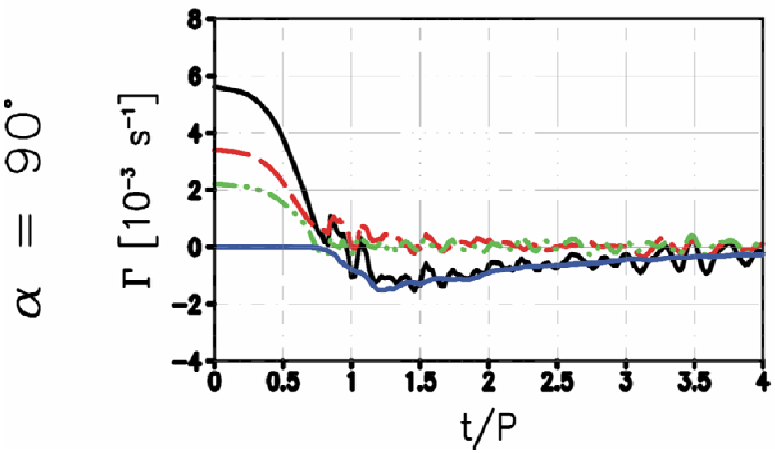
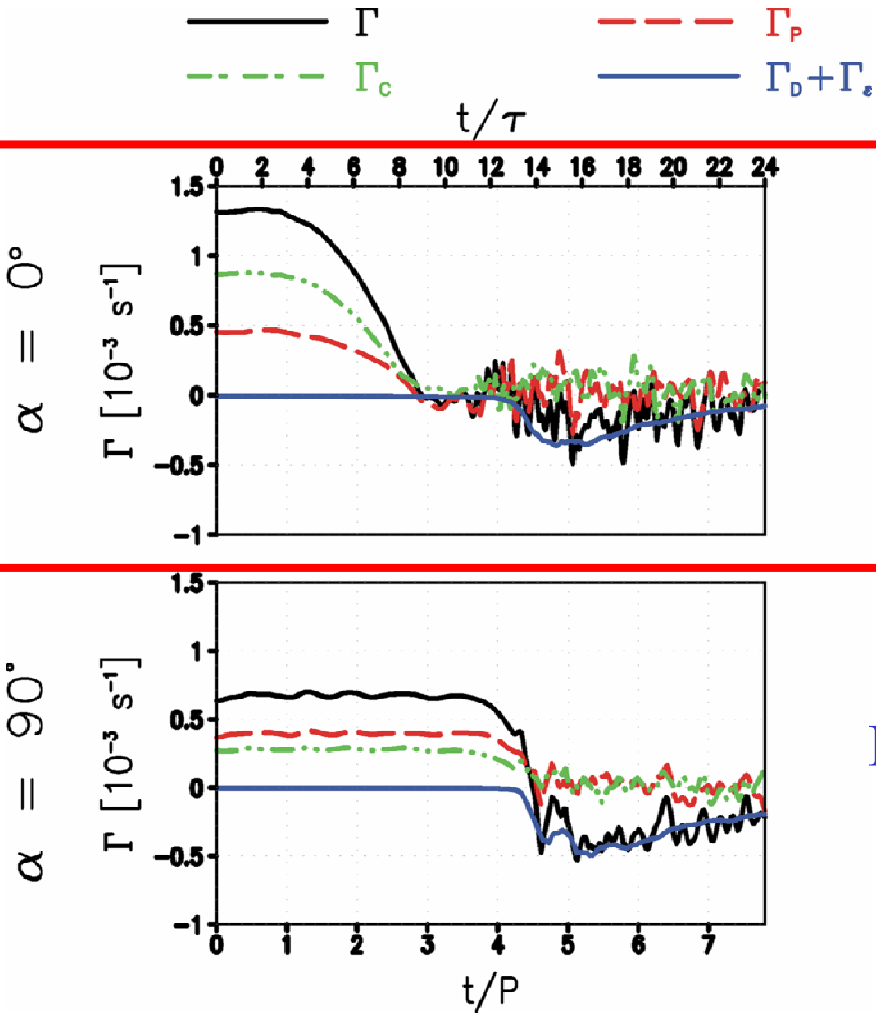


Strong growth perturbation energy:
shear instability

Energetics of a Breaking HGW with $a_0 < 1$

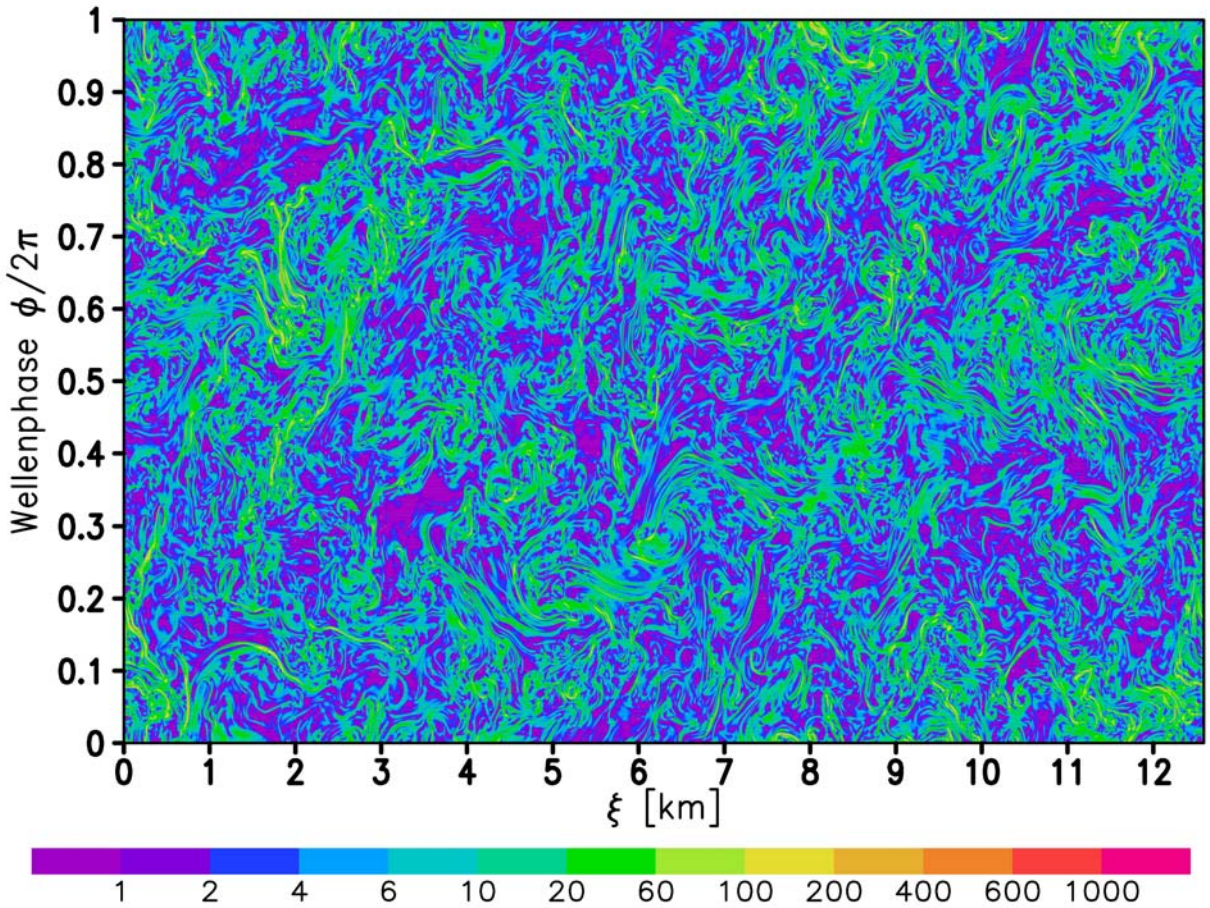


Strong growth perturbation energy
buoyancy instability

$$\Gamma = \frac{d\langle E' \rangle / dt}{2\langle E' \rangle} = \Gamma_P + \Gamma_C + \Gamma_D + \Gamma_\varepsilon$$

Breaking HGW with $a_0 < 1$: Dissipation rate

Dissipation rate (mW/kg) at ist maximum ($t = 5P$):



Summary

- ***the Richardson number is not a good criterion for turbulence from gravity waves!***
- **IGWs:** Breaking by SVs
- **HGWs:** Instabilities due to horizontal gradients
- **References:**
  - Achatz (2005, *Phys. Fluids*)
  - Achatz and Schmitz (2006a,b, *J.Atmos. Sci.)*
  - Achatz (2007a,b, *J. Atmos. Sci.*)
  - Achatz (2007c, *Adv. Space Res.*)
  - all PDFs available from my web page (http://www.geo.uni-frankfurt.de/iau/ThMet/english/Publications/index.html)
- **see also** Fritts et al. (2003, 2006, 2008a,b)